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Measuring and Analyzing Sovereign Risk with Contingent Claims

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Measuring and Analyzing Sovereign Risk with Contingent Claims

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Abstract

This paper develops a comprehensive new framework to measure and analyze sovereign risk. Since traditional macroeconomic vulnerability indicators and accounting-based measures do not address risk in a comprehensive and forward-looking way, the contingent claims approach is used to construct a marked-to-market balance sheet for the sovereign, and derive a set of credit-risk indicators that serve as a barometer of sovereign risk. Applications to 12 emerging market economies show the risk indicators to be robust and highly correlated with market spreads. The framework can help policymakers design risk mitigation strategies and rank policy options using a calibrated structural model unique to each economy.

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I. INTRODUCTION

As economies have become more reliant on private capital flows, they have also become more vulnerable to the volatility of capital flows, and to price and other shocks. A comprehensive framework is needed to analyze—and hopefully help prevent—large scale capital account crises and associated financial distress. A useful approach that has been gaining popularity since the Asian crisis is to assess the risk posed by potentially unstable positions in sectoral balance sheets, including in the corporate, financial, and public sectors. Shocks to interest rates, exchange rates, or market sentiment that bring about a deterioration in the value of a sector's assets compared to its liabilities lead to a reduction of its net worth. In the extreme case, net worth turns negative and the sector may become insolvent. In these cases, risks are transferred across balance sheets, triggering widespread distress. Risk transfer can be "bottom-up" from the corporate sector to the banking system and ultimately to the sovereign balance sheet, as was the case during the Asian crisis, or it can be "top-down," as was seen more recently in Latin America. Developing an effective approach to detect and assess balance sheet vulnerabilities before they become severe is essential to minimize risks and protect the stability of the overall economy. In this paper, the contingent claims approach (CCA) is used to measure and analyze risk on the public sector, or sovereign, balance sheet.[2] Estimating risk using such an approach has a long tradition in modern financial theory and has been widely applied in the analysis of corporate sector credit risk.[3] It is increasingly being used to estimate risk in the financial sector, but has yet to be broadly applied at the sovereign level.[4] This paper represents a first step in this direction.

Effective risk analysis must meet three objectives. First, it needs to identify existing balance sheet mismatches. Second, it must incorporate uncertainty inherent in balance sheet components since uncertain changes in future asset value relative to promised payments on debt obligations ultimately drive default risk. Third, effective risk analysis must translate uncertainty into quantifiable risk indicators that measure risk exposures to reveal whether balance sheet risks are building or subsiding. Such quantitative risk indicators should also incorporate forward-looking information.

The contingent claims approach meets all three objectives. It uses the basic structure of a balance sheet, adding market prices and uncertainty as key inputs, to derive simple risk indicators that are forward looking. In effect, this framework provides a marked-to-market balance sheet for the sovereign. In measuring sovereign risk, the contingent claims approach derives estimates for sovereign asset value and asset volatility—which are not directly

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[2] The contingent claims approach was applied to estimate balance sheet risk in the aggregated corporate sector in Gapen, Gray, Lim, and Xiao (2004). The analysis also provided estimates of risk transfer across the corporate, financial, and public sectors.

[3] Black and Scholes (1973), Merton (1973), and Merton (1998).

[4] Merton (1977); Gray (2002); Gray, Merton, and Bodie (2003); Draghi and Merton (2003); Gray (2004); and Chan Lau (2004).

observable—from the value and volatility of sovereign liabilities that are observable. These values are then weighed against existing contractual liabilities to provide a market-based assessment of sovereign default risk. Unlike traditional macroeconomic vulnerability indicators and accounting-based measures, which cannot address risk or uncertainty in a comprehensive or forward-looking manner and rely mainly on static ratios, the contingent claims approach provides a richer, dynamic way to measure and analyze risk.

This paper develops a set of key *credit risk indicators* to measure sovereign balance sheet risk. These include: distance to distress, probability of default, credit spreads, and the market value of risky foreign currency denominated debt. These indicators are closely related since they are all derived from the core contingent claim relationships. Associated with these risk indicators are *sensitivity measures* that report how responsive the credit risk indicators are to changes in underlying model parameters, such as changes in the value of sovereign assets and volatility. Importantly, the sensitivity measures capture nonlinear changes in value which are often observed during crisis periods.

To illustrate the usefulness of the credit risk indicators as a collective barometer of sovereign risk, they are subjected to robustness tests using observed market data for a sample of emerging market countries. The tests suggest a high degree of correlation between the credit risk indicators and the observed market data on spreads. As market credit spreads were not used as inputs in deriving the risk indicators, the high correlation suggests that the risk indicators can be confidently used as reasonable measures of sovereign credit risk, thus lending support to the contingent claims structural model developed in this paper. The risk indicators can be examined in individual country cases to evaluate whether market expectations of sovereign vulnerabilities are increasing or decreasing over time or they can be examined across countries to rank relative riskiness.

As a further demonstration of the applicability of the contingent claims approach in evaluating sovereign risk, the paper uses the model calibrated to market data to evaluate how risk indicators change given specific scenarios. Through scenario analysis, policymakers can observe the extent to which negative economic shocks could worsen sovereign financial soundness through capital outflows, a depreciating exchange rate or slower economic growth. As an additional step, Monte Carlo simulations are used in conjunction with the contingent claims approach to yield probability distributions and confidence intervals for the set of sovereign credit risk indicators. Since simulations allow for the assessment of many potential market scenarios, it provides for a more comprehensive risk analysis that includes probability distributions and value-at-risk (VaR) measures. Policymakers can use these tools to help them design and implement risk mitigation strategies to reduce balance sheet risk and to rank competing policy choices.

Finally, the paper points to two promising areas in which the contingent claims approach can be usefully applied: reserve management and debt sustainability. On reserve management, the contingent claims approach can be used to derive an appropriate target for reserve adequacy, where an adequate level of reserves could be defined as the level of reserves that keep the credit risk indicators above a specified threshold. On debt sustainability, the contingent claims approach offers several advantages over the traditional debt sustainability analysis which has tended to focus on ratios of debt-to-GDP as the primary criterion for deciding whether public debt is on a sustainable path. In particular, the approach provides a structural framework that relates debt payments with the capacity to pay, and threshold levels for sovereign credit risk.

The paper is structured as follows. Section II defines sovereign risk and introduces the contingent claims approach of measuring the value and volatility of sovereign assets. Section III constructs the contingent claims balance sheet. Section IV shows how the credit risk indicators are developed. Section V applies several robustness checks to the credit risk indicators to assess their correlation with actual market data. Section VI presents a calibrated baseline balance sheet for a hypothetical sovereign. Scenario and simulation analysis is conducted on the baseline balance sheet to assess the impact on sovereign risk. The section discusses briefly how this approach can be used to evaluate potential policy choices. Section VII details the next steps in the application of this approach to evaluating reserve management and debt sustainability. Section VIII concludes. Further details on the use of option pricing techniques to derive the credit risk indicators are provided in the appendix.

II. A Practical Approach to Sovereign Risk

Of the different types of sovereign risk, one of the most important is the risk of default. Sovereign default is often a culmination of accumulated distress, where the risk of sovereign default is effectively driven by the interplay of three main elements: the value of sovereign assets, asset volatility, and leverage. Sovereign asset value is defined as the combined *market value* of all sovereign assets. Asset value is an aggregate of different components which are dependent on the country's future economic prospects and policy decisions. Since future economic prospects are uncertain, asset volatility captures the inherent uncertainty, or variability, of future sovereign asset value. Leverage measures the size of the sovereign's contractual liabilities. Contractual liabilities are measured in *book value* terms since these are the amounts that the sovereign is obligated to pay.

The approach to sovereign risk outlined in this paper closely mirrors a similar process that has successfully been applied to estimate firm credit risk.[5] There are sufficient similarities between individual firm risk and sovereign risk to suggest a reasonable transfer of the contingent claims approach from corporate to sovereign risk analysis. These similarities are examined in more detail in Box 1. In the next section, the concept of sovereign distress is defined, and the interplay between the sovereign asset, its volatility and debt obligations in the determination of sovereign distress is discussed.

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[5] The contingent claims approach has been widely applied by financial market participants to measure the default probability of corporations and banks based on the market prices of the equity and book values of debt. See KMV (1993); Crosbie and Bohn (2001); Crouhy, Galai, and Mark (2000); and Cossin and Pirotte (2001) for the contingent claims approach to individual firm credit risk.

A. Defining Sovereign Distress: The Concept

Sovereign distress increases when the market value of sovereign assets declines relative to its contractual obligations on debt. Default ultimately occurs when the sovereign assets fall below the contractual liabilities. Contractual liabilities, therefore, constitute a *distress barrier, and* sovereign distress is measured by the relationship between sovereign assets relative to this distress barrier. Default risk increases when the value of sovereign assets declines towards the distress barrier or when asset volatility increases such that the value of sovereign assets becomes more uncertain and the probability of the value falling below the distress barrier becomes higher.[6]

Evidence from the universe of corporate defaults indicates that the market value of firm assets can sometimes trade below the book value of total liabilities for a significant period of time.[7] This is most often the case when the majority of liabilities are long-term, allowing the firm to continue servicing debt payments while undertaking steps to improve the financial health of the firm. A similar argument can be applied to sovereign credit risk, whereby the probability of distress is increased when most of the liabilities are short-term, or when rollover risk is highest. Therefore, the approach adopted in this paper follows the well-established procedure in estimating corporate default risk, namely that the value of sovereign assets that triggers an incidence of sovereign distress lies somewhere in-between the book value of total liabilities and short-term liabilities. This adjusted value of liabilities is defined as the distress barrier, and is commonly denoted as the sum of short-term debt, interest payments for one year, and half of long-term debt. [8]

The market value of sovereign assets in relation to the distress barrier is illustrated in Figure 1. The uncertainty in future sovereign asset value is represented by a probability distribution at the time horizon. At the end of the period, the value of sovereign assets may be above the distress barrier, indicating that debt service can be made, or below the distress barrier, leading to default. The probability that sovereign assets will fall below the distress barrier is simply the area of the distribution that lies below the distress barrier. The bottom panels in Figure 1 detail the effects of a decline in the value of sovereign assets and an increase in uncertainty over future sovereign asset value. In the first case, the lower expected

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[6] Volatility of sovereign assets can differ across countries for many reasons, including, but not limited to, the level of international reserves on the government's balance sheet, the exchange rate, and variations in government revenue and expenditures. Countries with lower asset volatility are generally able to use larger amounts of leverage with relative comfort while countries with higher asset volatility would be better-off taking on less leverage.

[7] Crosbie and Bohn (2003). Moody's KMV maintains a database with over 250,000 company-years of data and 4,700 incidents of default or bankruptcy.

[8] This definition of the distress barrier is identical to that used by Moody's KMV in corporate sector default risk analysis (Crosbie and Bohn, 2003). Short-term is defined as one year or less by residual maturity.

sovereign asset value means more of the probability distribution lies below the unchanged distress barrier, which results in a higher probability of default. In the second case, the lower expected sovereign asset value is also accompanied by an increase in asset volatility. The increased volatility widens the probability distribution, leading to an even higher probability of default as more of the area under the probability distribution now lies below the distress barrier.

B. Estimating the Value of Sovereign Assets

Given the conceptual definition of sovereign distress, how does one go about estimating it? The main challenge would be deriving an accurate estimate for the market value and volatility of sovereign assets. While the levels and amounts of contractual liabilities are relatively easy to determine from balance sheet information, the same is not true when measuring the value of sovereign assets or its volatility.[9] The market value of sovereign assets is not directly observable and must therefore be estimated. With this in mind, there are several ways to value an asset:

- Determining value from observed market prices of all or part of the asset. This can be from a market price quote, direct observation, bid-ask quote or other similar direct measures;

- Determining value by a comparable or adjusted comparable. A sophisticated version of obtaining a comparable value is the present value of a discounted expected cash flows—such as the primary surplus—with an appropriate discount rate;

- Determining value from an implied value where the balance sheet relationships between assets and liabilities allow the observed prices of liabilities to be used to obtain the implied value of the assets.

The three methods have different advantages and disadvantages. The first method is straightforward but difficult to apply because only a few components of sovereign assets have directly observable market prices. International reserves are both observable and have a market value, yet the remaining items lack observable market prices. [10] The second method using comparables is commonly used, but also has shortcomings. These are related to the difficultly of projecting future cash flows, deciding the appropriate discount rate, and determining all of the relevant components that underlie the cash flow projections for

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[9] Foreign currency debt in global markets is predominantly fixed-rate, “bullet” maturity debt which results in easily defined contractual flows. Some global debt is amortizing, but these payments are usually well-specified. The main difficulties in estimating debt payments arise when the debt payments are linked to changes in interest rates, exchange rates, or inflation. These forms are more often found in domestic as opposed to global capital markets.

[10] Buiter (1993) discusses in detail the many items on the balance sheet of the public sector, including nonmarketable items, such as social overhead capital.

tangible and intangible items included in the asset value estimation. For example, determining the present value of the net fiscal asset requires estimates of future economic performance, the political commitment to a variety of programs including social security and other entitlement programs, and the use of an appropriate discount rate. Estimates for the value of other assets like the value of the public sector monopoly on money issuance run into similar problems. Furthermore, it is unclear how asset volatility should be best measured under the first two methods.

The third method, which is the approach adopted in this paper, circumvents the problems in the first two methods by estimating sovereign asset value and volatility indirectly with information on observable values of the liability side of the balance sheet. This approach relies on the relationship between assets and liabilities. Since liabilities are claims on current or future assets, this approach is often referred to as "contingent claims" analysis and yields an "implied" estimate for sovereign assets. The calculation of implied values is a very common technique in the finance world. The collective view of many market participants is incorporated in the observable market prices of liabilities and the change in the market price of these liabilities will determine its volatility. This contingent claim approach implicitly assumes that market participants' views on prices incorporate forward-looking information about the future economic prospects of the sovereign. This does not imply that the market is always right about its assessment of sovereign risk, but that it reflects the best available collective forecast of the expectations of market participants.

Implementing contingent claims analysis to derive the implied sovereign asset value and volatility requires several steps and assumptions. These are discussed in the next section.

III. CONTINGENT CLAIMS ANALYSIS OF THE SOVEREIGN BALANCE SHEET

The contingent claims sovereign balance sheet is constructed from the basic accounting balance sheet of the government and monetary authorities. Figure 2 shows the balance sheets of the government and monetary authorities as two segregated yet linked balance sheets. Government liabilities include foreign currency debt, domestic currency debt, and obligations owed by the government to the monetary authorities and the guarantees to "too-important-to-fail entities." Government assets include a claim on a portion of the foreign currency reserves held by the monetary authority and other public sector assets such as the present value of the primary fiscal surplus. The balance sheet of the monetary authority in Figure 2 has assets consisting of international reserves (net foreign assets) and credit to government (net domestic assets). Liabilities of the monetary authority are base money and a claim of the government on a portion of foreign currency reserves.

In order to use the contingent claims balance sheet to estimate the asset and volatility of sovereign assets, three steps are needed:

- First, a sovereign balance sheet need to be constructed with the liability side containing only elements with observable quantities and market prices, and in a common currency.

- Second, assumptions on the seniority of sovereign liabilities need to be defined to use standard contingent claim relationships.

- Third, option pricing techniques are used to estimate the value and volatility of sovereign assets from the observable market value and volatility of sovereign liabilities.

A. Consolidating the Sovereign Balance Sheet

The two segregated balance sheets of the government need to be consolidated so that every entry on the liability side can be traced to observable data and the entire balance sheet is denominated in a common currency. Balance sheets for the country case studies presented in this paper are measured in U.S. dollars for ease of comparison, but the analysis holds even if they are valued in domestic currency.[11] Through the consolidation process the government claim on foreign currency reserves and credit to government net out and guarantees to too-important-to-fail entities are subtracted from the sovereign asset.[12] Figure 3 shows the consolidated sovereign balance sheet denominated in a common foreign currency. All the entries on the liability side of the contingent claim sovereign balance sheet in Figure 3 are now directly observable from market prices.

B. Seniority of Consolidated Balance Sheet Liabilities

Seniority of sovereign liabilities is not defined through legal status as in the corporate sector, but may be inferred from examining the behavior of government policymakers during periods of stress. In times of stress, governments often make strenuous efforts to remain current on their foreign currency debt, efforts that effectively make such debt senior to domestic currency liabilities.[13] The payment of foreign currency debt requires the acquisition

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[11] Measuring the balance sheet in U.S. dollars results in variable sovereign assets versus a fixed distress barrier. Measuring the balance sheet in domestic currency will result in both variable sovereign assets and a variable distress barrier. In either configuration, the contingent claim formulas will produce the same results.

[12] The implicit guarantees to the financial sector, or other entities, could remain on the liability side of the consolidated public sector balance sheet and modeled as implicit put options. For more details see Merton (1977); Gray, Merton, and Bodie (2002 and 2003); Gapen, Gray, Lim, and Xiao (2004); and Van den End (2005). These papers link the sovereign to the contingent claim balance sheets of the banking or corporate sectors. The detailed analysis of the links to other sectors is beyond the scope of this paper.

[13] Support for viewing foreign currency debt as senior can be found in the literature on "original sin" in Eichengreen and others (2002). Support for modeling domestic currency liabilities as junior claims can be found in Sims (1999) who argues that local currency debt has many similarities to equity issued by firms. He models domestic currency debt as "equity" and in this setting, domestic currency debt becomes an important absorber of fiscal risk, just as equity is a cushion and risk absorber for firms. As long as there is some probability that the government will run a primary surplus in the future and/or will engage in the repurchase of domestic currency debt then such debt has value.

of foreign currency, which the government has a more limited capacity to produce. In contrast, the government has much more flexibility to issue, repurchase and restructure local currency debt. For this reason, governments sometimes introduce capital controls to prevent convertibility and preserve remaining international reserves to service sovereign external debt obligations. In other instances, governments have insisted on the mandatory rollover or restructuring of domestic currency debt during periods of distress without simultaneously engaging foreign currency creditors. In these circumstances, holders of domestic currency liabilities will see the value of their claim greatly reduced since sovereign distress are often accompanied by instances of exchange rate depreciation, which reduces the value of the domestic currency liabilities in terms of its value in foreign currency.

Three recent examples of sovereign debt restructuring illustrate this implicit seniority structure. Russia in 1998–99 introduced capital controls, forced a lengthening of maturities on domestic currency government debt, and declared a unilateral moratorium on private sector external debt obligations while still publicly stating their intention to honor sovereign external debt.[14] In March 1999, Ecuador froze all checking, savings, and time deposits to limit further exchange rate depreciation.[15] In August 1998, Ukraine imposed convertibility restrictions in the foreign exchange market and selectively restructured domestic debt held by banks.[16] (Other examples include government restructuring of debt held by domestic banks or pension funds, thereby reducing their present value, prior to the restructuring of foreign currency denominated external debt).

For these reasons, this paper models foreign currency debt as a senior claim and domestic currency liabilities as junior claims.[17] Default in this paper, therefore, means default on foreign currency debt and the distress barrier, *DB*, defines the level at which payments on foreign currency debt cannot be made. The distress barrier is assumed to equal to the book value of short-term external debt plus interest and one-half of long-term external debt.[18] Default is assumed to occur when the value of sovereign assets declines below the distress barrier. As the junior claim, the value of domestic currency liabilities is dependent on the level of sovereign assets above and beyond what is necessary to service senior foreign currency debt. Senior claims, however, are risky because asset value may not be sufficient to

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[14] Ariyoshi and others (2000).

[15] Gule and others (2003); Allen (2002).

[16] Shadman-Valavi (1999); Allen (2002).

[17]It should be noted that this ordering can be flexible and the contingent claims framework can be adapted to any number of different seniority structures. In future work, the seniority assumption will be relaxed to take into account multiple layers of liabilities, described in more detail in the appendix.

[18] An alternative procedure would be to use all of short-term external debt and interest, plus long-term external debt discounted by the risk-free rate.

meet promised payments. The value of senior claims, therefore, can be seen as having two components, the *default-free value* (promised payment value) and the *expected loss* associated with default when the assets are insufficient to meet the promised payments. The value of junior claims is the residual value of sovereign assets after the promised payments to senior claims have been made. Thus, in financial terminology, the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, while the value of *risky* foreign currency debt can be modeled as default-free value of debt—equivalent to the distress barrier—minus the expected loss in the event of default.

The next section discusses how changes in the values of observed market variables—the value and volatility of sovereign liabilities—are used to infer changes in unobserved variables—the value of sovereign assets based on the contingent claims balance sheet and seniority structure above.

C. Calculating Implied Sovereign Asset Value and Volatility

Since the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, standard option pricing techniques can be applied to derive implied estimates for sovereign asset value and volatility. The balance sheet states that the value of assets is the sum of the value of domestic and foreign currency liabilities, $V_A = V_{DCL} + V_{FCL}$, where V_{FCL} represents the value of risky foreign currency liabilities and V_{DCL} represents the value of domestic currency liabilities.[19] The option pricing formulas employed to estimate sovereign asset value and volatility rely on a few select variables: the value and volatility of domestic currency liabilities (V_{DCL} and Vol_{DCL}, respectively), the distress barrier (DB), the risk-free interest rate (r_f), and time (t). As shown in more detail in the appendix, these variables can be combined into two equations,

$$V_{DCL} = Function\,(\,\boldsymbol{V_A}, \boldsymbol{Vol_A}, DB, t, r_f\,), \qquad (1)$$

$$Vol_{DCL} = Function\,(\,\boldsymbol{V_A}, \boldsymbol{Vol_A}, DB, t, r_f\,), \qquad (2)$$

which can be solved simultaneously to derive the two unknowns, which are the implied market value (V_A) and volatility (Vol_A) of sovereign assets. Thus the information embedded in the value and volatility of domestic currency liabilities (in units of foreign currency) and the distress barrier derived from the book value of foreign currency debt yield estimates of implied sovereign asset value and implied asset volatility over a given time horizon. The volatility of the domestic currency liabilities (domestic currency debt and base money) come from a variety of sources, including the volatility of the exchange rate and of the quantities issued. The volatility of the exchange rate process is relatively more important in a floating

[19] The value of senior foreign currency liabilities can also be obtained using the implicit put option in risky debt (Gray, Merton, Bodie 2003; Gapen et al., 2004; Gray 2004).

exchange rate environment while the quantities of domestic currency liabilities may vary substantially under a fixed or heavily managed exchange rate system.

IV. Sovereign Credit Risk Indicators

Having derived the estimates of implied asset value and volatility, this section details how they can be used to develop useful indicators of sovereign risk. These risk indicators are the distance to distress, the risk-neutral probability of default, the value of senior foreign currency debt and the sovereign credit risk premium or sovereign risk-neutral credit spread.[20] While price and spread information may be easily observable from the market, the market information itself does not reveal the rationale underlying the risk premium nor does it reveal what is often the most valuable piece of information in risk analysis—how much risk exposures could change as the health of the sovereign improves or declines on the margin. The contingent claims approach links the credit risk premium to the balance sheet framework, allowing for an evaluation of the structural determinants of credit risk.

1. Distance to Distress and Risk-Neutral Probability of Default

The implied value and volatility of sovereign assets can be combined with the distress barrier to produce an indicator of default risk, referred to here as the distance to distress. This measure computes the difference between the implied forward market value of sovereign assets and the distress barrier scaled by a one standard deviation move in sovereign assets. The distance to distress is defined conceptually as:

$$\frac{\textit{Implied market value of sovereign assets} - \textit{Distress barrier}}{\textit{Implied market value of sovereign assets} * \textit{Sovereign asset volatility}}.$$

The numerator above measures the distance between the expected one-year ahead market value of sovereign assets and the distress barrier. This amount is then scaled by a one-standard deviation move in sovereign assets. The distance to distress therefore yields the number of standard deviations sovereign asset value is from distress. Lower market value of sovereign assets, higher levels of foreign currency debt, and higher levels of sovereign asset volatility all serve to decrease the distance to distress.

In formula representation,

[20] Risk-neutral valuation is an important factor underlying the derivation of the Black-Scholes option pricing formula whereby the value of the option can be derived by forming a riskless hedge portfolio. Thus, option values do not depend on the investor’s or decision maker’s attitude toward risk, which is a major benefit of this approach. Alternative balance sheet approaches based on discounted cash flows are subject to serious error not only from errors in cash flow projections, but from errors in choosing the discount rate. See Hull (1993, pp. 221–222) and Chriss (1997, pp.190–193) for additional discussion of risk-neutral valuation.

$$\text{distance to distress} = d_2 = Function\ (\ V_A, DB, r_f, Vol_A, t\ ), \tag{3}$$

where d_2 is from the Black-Scholes option pricing formula (see appendix). Distance to distress for a hypothetical sovereign is illustrated in Figure 1.

The option pricing relationships in equations (1) and (2) above also yield a measure of probability of default, commonly referred to as the risk-neutral default probability. The probability of default is simply the probability that future sovereign asset value will fall below the distress barrier. The option pricing formula used in this analysis assume that future sovereign asset value is distributed log-normally and the risk-neutral probability of default is therefore the shaded area that lies below the distress barrier as shown in Figure 1. The risk-neutral default probability (RNDP) is,

$$RNDP = N(-d_2), \text{ where } d_2 = Function(\ V_A, DB, r_f, Vol_A, t\ ), \tag{4}$$

and $N(.)$ is the cumulative normal distribution at the distance to distress, d_2.

2. Value of Foreign Currency Liabilities and Sovereign Credit Risk Premium

The other two useful sovereign risk indicators that can be obtained using the contingent claims approach are the sovereign credit spread or credit risk premium, and the market value of foreign currency liabilities. The value of risky senior foreign currency liabilities (V_{FCL}) can be derived using the implied value and volatility of sovereign assets, equations (1) and (2), and the balance sheet identity noted above, that is, the value of assets is the sum of the value of domestic and foreign currency liabilities. Using these relationships together yields the value of risky foreign currency liabilities,[21]

$$V_{FCL} = function\left(\left(\frac{V_A}{DBe^{-r_f t}}\right), Vol_A, t, r_f\right). \tag{5}$$

The term, $DBe^{-r_f t}$, is the distress barrier discounted to the present by the risk-free rate over the time horizon t. Since the distress barrier is based on the book value of foreign currency debt, it is equivalent to the default-free value of foreign currency liabilities. If the ratio of sovereign assets to the default-free value of foreign currency liabilities rises or the volatility of sovereign assets declines, the value of risky foreign currency liabilities increases. Conversely, as the ratio of sovereign assets over the discounted distress barrier falls or asset volatility rises, the market value of risky debt will decline, possibly falling below its default-free value. In other words, if the sovereign becomes more wealthy and the stream of its income less uncertain, the market value of its foreign currency debt will become more

[21] See appendix.

valuable, and vice versa. Therefore, the value of foreign currency liabilities is a useful indicator of the expected gain/loss in asset value that the sovereign is likely to experience if the market prices of its debt increase/decrease.

Manipulating equation (5) results in an estimate of the risk neutral credit spread (RNS) of,

$$RNS = y - r_f = function\left(\left(\frac{V_A}{DBe^{-r_f t}}\right), Vol_A, t, r_f\right), \qquad (6)$$

where $y = -(1/t)\ln(V_{FCL}/DB)$. The left hand side of equation (6) represents the yield to maturity on risky foreign currency debt less the risk-free rate of interest and is therefore equivalent to a credit risk premium, or risk-neutral credit spread. In addition to the risk-free rate and time, the sovereign risk premium is a function of only two variables: the volatility of sovereign assets and the ratio of the value of sovereign assets to the distress barrier. Both increases in the ratio of sovereign assets to foreign currency liabilities and decreases in sovereign asset volatility reduce the sovereign risk premium. Conversely, as the ratio of sovereign assets to foreign currency liabilities decreases or sovereign asset volatility increases, the risk premium widens. The intuition is similar as before. The sovereign's credit risk declines if it has a cushion of assets to protect it from negative shocks and the cushion is relatively stable.

It is useful to note that no market information on foreign currency denominated debt, namely bond spreads or credit default swap spreads, have been used while computing the value of risky foreign currency liabilities and credit risk premium in the model. Only information on the book value of payments on existing foreign currency debt is used in construction of the distress barrier. This is combined with market information from domestic currency liabilities and the exchange rate to estimate the value of foreign currency liabilities and the credit risk premium. This is noteworthy since the model output can be then compared with readily available market information to evaluate the robustness of this approach.

3. Sensitivity Measures

Associated with the sovereign risk indicators are sensitivity measures, which reveal how responsive the set of risk indicators are to changes in model parameters, namely changes in the value of sovereign assets and asset volatility. This paper focuses on eight relevant sensitivity measures. The first four are the changes in distance to distress, risk-neutral default probability, risk-neutral credit spreads, and value of foreign currency debt from a 1 percent change in the value of sovereign assets. The second four are changes in the same risk indicators from a 1 percent change in sovereign asset volatility. These sensitivity measures are critical in risk analysis because they capture nonlinear changes in value, and equally important, they look beyond the current level of distance to distress, spreads or probability of default. In other words, they provide an indication of the potential *risk exposure* of the sovereign. The sensitivity measures are highest when sovereign asset value is in the neighborhood of the distress barrier, reflecting magnified default risk. In this instance, small changes in underlying asset value in either direction will have proportionately larger impacts on the balance sheet risk indicators. In sum, while the credit risk indicators yield current estimates of sovereign balance sheet risk, the sensitivity measures point to how sovereign risk could further change if the balance sheet improves or weakens on the margin.

V. Robustness of Sovereign Credit Risk Indicators

The degree to which the contingent claims risk indicators closely parallel actual market data will indicate their usefulness as early warning indicators of sovereign risk. To this end, a historical time series of the risk indicators is compared with actual market data for a number of emerging market countries. The historical data for the sovereign risk indicators in equations (3) – (6) were obtained from the Macrofinancial Risk (M*f*Risk) model while the historical market data were obtained from credit default swap and external debt markets.[22] Robustness of the sovereign credit risk indicators is examined through their correlation and relationship with actual data and through regression analysis.

A. Correlation with Market Data

If the model output is robust, distance to distress should be negatively correlated with actual sovereign credit spreads. As distance to distress increases, credit risk should decline and be reflected in lower credit default swap spreads. Figure 4 displays the relationship between the distance to distress indicator for twelve emerging market sovereign balance sheets versus that country's observed credit default swap (CDS) spread.[23] Table 1 reports the correlation of the risk indicators with the observed sovereign credit default swap spreads and EMBI spreads.[24] As can be seen there is a very high correlation for most countries between the two risk indicators—distance to distress and risk-neutral spread—and the observed CDS spreads and EMBI+ spreads from January 2003 to August 2004. The reported correlations confirm the expected negative relationship between distance to distress and both CDS and EMBI+ spreads. The correlations also display a high degree of significance as 29 of the 34 reported correlations between distance to distress and CDS spreads are significant at the 95 or 99 percent level. In many cases, correlation is highest with the 5-year CDS spread that likely reflects the greater liquidity in this market relative to the shorter maturity CDS market. A

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[22] The M*f*Risk model was developed by Macro Financial Risk, Inc. and applied to 17 countries under a joint research effort between Moody's and Macro Financial Risk, Inc. Access to M*f*Risk is only available through subscription.

[23] In Figure 4 the distance to distress scale is inverted. While this visually depicts a positive correlation between distance to distress and credit spreads, this implies that increases in distance to distress should result in lower spreads on credit default swaps.

[24] The reported correlations in Table 1 were computed using Spearman's rank correlation instead of conventional correlation. Conventional correlation is inappropriate in this case since it implicitly assumes linear relationships among variables, an assumption which contradicts the nonlinear relationship between variables as found in this paper. Spearman's rank correlation is a less restrictive measure to gauge relationships among variables since it does not impose any linearity assumptions.

similar level of significance is found between distance to distress and country EMBI+ spreads with 8 of the 9 reported correlations significant at the 99 percent level.

As a second check on robustness, the risk-neutral sovereign credit spread for each country is compared with the EMBI+ spread and CDS spread.[25] Figure 5 displays the expected positive relationship between the risk-neutral sovereign credit spread and each EMBI+ country spread for nine emerging markets for the sample period from January 2003 to August 2004. The correlation between the risk-neutral sovereign credit spread and the respective EMBI+ spread during the same time period is also reported in Table 1. The correlations show the expected positive relationship between the risk-neutral credit spread and EMBI+ country and CDS spreads. The correlations between the risk neutral credit spread and the CDS spread display a high degree of significance at the 95 and 99 percent levels for 30 out of 34 reported correlations. The correlations between the risk-neutral credit spread and EMBI+ spread display significance at similar levels in 8 out of 9 cases.

B. Regression Analysis

Two fixed effects panel regressions are used to estimate the relationship between risk-neutral spreads and EMBI+ country spreads and CDS spreads. The mapping from risk neutral spreads to actual market spreads is important because the risk neutral spreads tend to underestimate actual market credit spreads. The fixed effects model treats differences across countries in the sample as parametric shifts of the regression function (i.e., differences across countries are captured in differences in the constant term). This approach yields the following relationships:

- **Risk-neutral credit spreads and observed CDS spreads.** The relationship between risk-neutral credit spreads and observed CDS spreads is estimated by applying a fixed effects panel regression to a combined cross-country sample of 981 observations from April 2002 to August 2004.[26] Results, which are reported in Table 2, indicate that the coefficient and constants are highly significant at all confidence intervals and the R-squared from the panel regression is 88 percent.

- **Risk-neutral credit spreads and EMBI+ spreads.** The relationship between risk-neutral credit spreads and EMBI+ spreads is estimated by applying a similar fixed effects panel regression to the same cross-country sample of 981 observations from April 2002 to August 2004. Results, which are reported in Table 3, indicate that the coefficient and constants are highly significant at all confidence intervals and the R-squared from the panel regression is 96 percent.

[25] The EMBIG index has replaced the EMBI+ index as the preferred index for tracking emerging market credit spreads, but historical EMBIG index data was not available at the time of the writing of this paper.

[26] The countries in the sample include Brazil, Bulgaria, Colombia, Malaysia, Mexico, the Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

Given the goodness of fit of the above regressions, the individual country panel equations can be used to map sovereign risk-neutral credit spreads into: (i) actual CDS spreads, and (ii) actual EMBI+ spreads. For example, the estimated equation for Mexico used to map sovereign risk-neutral credit spread into the actual spread on credit default swaps is,

$$\ln(CDS_t) = 1.72 + 0.52 * \ln(RNS_t) . \qquad (7)$$

The similar estimated equation used to map risk-neutral credit spreads into actual EMBI+ spreads for Mexico is,

$$\ln(EMBI_t) = 4.78 + 0.15 * \ln(RNS_t) . \qquad (8)$$

As a numerical example, suppose that application of equation (6) results in risk-neutral spreads on foreign currency debt for Mexico of 200 basis points. Inserting this value into equations (7) and (8) results in a credit default swap spread of 88 basis points and an EMBI+ spread of 263 basis points.

The relationship between sovereign risk-neutral default probability and estimated actual default probability can also be determined. This procedure is necessary since risk-neutral default probabilities overstate the actual probability of default. To implement this comparison, some estimate of actual default probability is needed. In the application of the contingent claims approach to corporate credit risk, the standard adjustment mechanism is to map firm risk-neutral default probabilities against a database of actual corporate defaults.[27] However, a sufficiently large dataset of sovereign defaults is not available, meaning some other approach is necessary. A second best approach is to use estimates of actual default probability, or market implied default probabilities (MIDP), which can be obtained from credit default swap spreads assuming a specific loss given default and time horizon (a recovery rate of 30 percent was used in this analysis).[28] Using this approach, a fixed effects panel regression is applied to a cross-country sample of 935 observations from January 2003 to August 2004 in order to examine the relationship between risk-neutral default probabilities

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[27] For example, Moody's KMV (MKMV) utilizes 30 years of historical data over 6,000 public and 70,000 private company default events to derive the firm specific probability of default, which is referred to as the Expected Default Frequency™ (EDF).

[28] Market implied default probabilities (MIDP) can be obtained from CDS spreads through the following equation:

$$MIDP = \frac{1 - \exp(-spread * t)}{1 - R},$$

where *spread* is the net 1-year credit default swap spread, t is the time horizon (equal to 1 in this case), and R = 30 percent is the recovery rate. If the 1-year CDS spread is 180 basis points, the implied default probability is 2.5 percent. See appendix.

and market implied default probabilities.[29] The fixed effects panel regression displays high explanatory power with an R-squared of 93 percent. Results are reported in Table 4.

However, there is a problem with the panel regression results. As can be seen through close examination of Table 4, the regression equations for Korea, Malaysia, Mexico, Poland, and South Africa result in market implied default probabilities that are higher than risk-neutral default probability, which is a contradiction. This problematic result is likely due to two factors: (i) the assumption of a constant loss given default for all countries regardless of credit risk; and (ii) lack of a sufficiently long time series for credit default swaps. In practice, loss given default may change as probability of default and credit default swap spreads change. More sophisticated methods of estimating MIDP from credit default swap data are therefore needed, including methods that allow the recovery rate to vary with probability of default. These advanced methods are beyond the scope of this paper and are left for future research.

An alternative approach is to pool the individual country data into one regression to increase the number of observations, while maintaining the assumption of a constant loss given default.[30] The estimated equation using data from the pooled countries in the sample is,

$$\ln(MIDP_t) = \underset{(-15.54)}{-1.24} + \underset{(23.23)}{1.01} * \ln(RNDP_t) \qquad R^2 = 0.735, \tag{9}$$

where the numbers in parenthesis represent the relevant t-statistic. While the explanatory power of this pooled regression falls slightly relative to the panel regression reported in Table 4 (R-squared declines from 0.93 to 0.74), the level of explanatory power remains high and the relationship is highly significant. Furthermore, equation (9) produces a market implied probability of default that is lower than the risk-neutral probability of default. For example, suppose that application of equation (4) results in a risk-neutral probability of default equal to 8 percent. Inserting this value into equation (9) results in a market implied default probability of 2.3 percent. In other words, actual probability of default is approximately one-third of risk-neutral probability of default.

The sensitivity of sovereign spreads in response to a change in the distance to distress is a nonlinear relationship. Figure 6 plots observed market spreads (one-year CDS spreads) versus model distance to distress for all twelve countries in the sample (889 data pairs from the period mid-2002 to mid-2004). The figure shows the relationship aggregated across countries. The solid line is a best fit regression, with an R-squared of 0.80. It shows the spread going up exponentially as the distance to distress declines. The sensitivity of changes in spreads, for a given change in distance to distress, is much lower for countries with a high

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[29] The countries in the sample include Brazil, Colombia, Korea, Malaysia, Mexico, the Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

[30] Crouhy, Galai, and Mark (2000) also assume constant loss given default.

distance to distress. As the distance to distress declines from 1.5 to 1.4, the spread increases on average by 35 basis points. However, if the distance to distress drops from 0.5 to 0.4 the spread increases on average by 375 basis points.

The robustness checks in this section suggest that the distance to distress, risk-neutral credit spread, and risk-neutral probability of default are useful for evaluating sovereign vulnerabilities. The evidence indicates that the book value of foreign currency liabilities along with market information from domestic currency liabilities and the exchange rate contain important information about changes in the value of foreign currency liabilities and credit risk premium. The nonlinearities and inclusion of volatility in the option pricing relationship used in this analysis contributes to the high degree of explanatory power and correlation with actual data. Finally, the estimated relationships in equations (7) – (9) allow for straightforward transformation of model outputs into estimates of observable market data.

VI. Scenario and Simulation Analysis: Hypothetical Sovereign

With robustness verified, the structural models calibrated using the contingent claims framework and unique to each economy can be used with scenario and simulation analysis to evaluate shocks and policies. The goal is to estimate the potential effects of changes in economic conditions and impact of government policies on sovereign credit risk and sensitivity indicators. To begin with, a baseline balance sheet for a hypothetical sovereign is calibrated and the resulting baseline risk indicators and sensitivity measures are reported (Table 6). Scenario analysis is then conducted using two capital flow examples and the resulting point estimates for the credit risk indicators and sensitivity measures are compared to the baseline set of indicators. Next, the scenario analysis is extended using Monte Carlo simulations, which provides one way of generating a large number of market outcomes and produces probability distributions for each risk indicator. Unlike the scenario analysis which is intended to investigate the effects of a specific market outcome, the Monte Carlo simulation draws randomly from sample interest rate and exchange rate distributions to compute probability distributions and confidence intervals for a set of market outcomes. This process allows for the “stress testing” of the risk levels of the sovereign risk indicators to derive what are commonly known as value-at-risk (VaR) measures.

A. The Baseline

The starting point is the baseline balance sheet as displayed in Table 5:

- *Calibrated values.* The distress barrier is assumed to be US$100 billion, comprising short-term foreign currency debt plus interest of US$40 billion and one-half of long-term debt of US$60 billion. The value and volatility of domestic currency liabilities in dollar terms are US$82 billion and 0.76 (76 percent), respectively. Using equations (1) and (2), the implied value of sovereign assets is US$175 billion and the implied volatility of sovereign assets is 0.38 (38 percent). Foreign currency reserves are assumed to make up US$40 billion of implied sovereign assets.

- *Credit risk indicators*. From equation (3), the resulting distance to distress is 1.4 standard deviations. The distance to distress results in a risk-neutral probability of default of 8 percent in equation (4). Equations (5) and (6) yield the value of risky foreign currency debt as US$95 billion and risk-neutral credit spread of 115 basis points, respectively.[31] The market value of risky foreign currency debt implies a present value expected loss of US$1 billion. This value is derived from the difference between the discounted present value of the distress barrier (using a risk-free rate of 4 percent yields a present value distress barrier of US$96 billion) and the implied market value of foreign currency debt.

- *Sensitivity measures*. Sensitivity measures are calculated from a 1 percent change in sovereign asset value and volatility. For example, when the value of sovereign assets decreases by 1 percent, Table 5 shows that the distance to distress falls by 0.03 standard deviations (i.e., from 1.4 to 1.37 standard deviations), risk-neutral default probability increases by 0.41 percent, risk-neutral credit spreads increase by 7 basis points, and the expected loss on foreign currency debt increases by US$70 million. Sensitivity measures are also reported for a 1 percent change in sovereign asset volatility.

As previously discussed, the sensitivity measures capture the nonlinearity present within the contingent claims relationships. An example of the presence of nonlinearities is shown in Figure 6, which plots how risk-neutral spreads change in response to changes in sovereign asset value. As the value of sovereign assets approach the distress barrier, a 1 percent reduction in sovereign assets results in a 25 basis point increase in credit spreads compared with the baseline calibration, where the same 1 percent reduction in sovereign asset value only leads to a 7 basis point increase in credit spreads. The nonlinearity implies that the drop in sovereign assets has a proportionately larger impact on credit risk when sovereign assets are close to the distress barrier. The converse is true when the implied value of sovereign assets is well-above the distress barrier.

B. Scenario Analysis

Two scenarios are examined and compared with the baseline. Scenario 1 represents the potential negative effects associated with capital outflows and Scenario 2 illustrates the positive effects from capital inflows. First, suppose that economic conditions deteriorate so that capital outflows occur. Capital outflows are normally associated with some combination of an exchange rate depreciation, a drop in domestic debt prices (possibly associated with a rise in domestic interest rates), and an increase in volatility of both debt prices and the exchange rate. The impact of capital outflows on the sovereign balance sheet risk indicators depends in part on the response of policymakers. The assumption in this example is that policymakers accommodate some, but not all, of the shock. This would include some loss of

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[31] At this point, the risk-neutral default probability and spread would then be mapped into market implied probability of default and market spreads using equations (7) – (9), but this step is ignored here since this exercise is only for a hypothetical sovereign balance sheet.

international reserves, tighter interest rate policy, and increase in the net fiscal asset. Under this scenario, sovereign asset value is assumed to fall by US$25 billion to US$155 billion, with international reserves falling from US$40 to US$35 billion. Sovereign asset volatility increases from 38 percent to 43 percent. The left column in Table 6 (Scenario 1) displays the new contingent claim sovereign balance sheet, balance sheet risk indicators, and sensitivities after capital outflows.

In sum, capital outflows worsen the credit risk indicators and risk exposure of the sovereign has increased relative to the baseline. Distance to distress falls from 1.4 to 1.0 standard deviations and risk-neutral probability of default increases from 8 to 17 percent. Risk-neutral spreads on foreign currency debt rise to reflect the increased risk of nonrepayment as the expected loss has increased from US$1.5 billion to US$3 billion. In addition to a worsening of the credit risk indicators, the sensitivity measures have increased since implied sovereign asset value is fewer standard deviations from the distress barrier. For example, a 1 percent decline in sovereign asset value under the baseline scenario increased risk-neutral default probability by 0.41 (from 8 to 8.41 percent) and risk-neutral spreads by 7 basis points, while in this capital outflow scenario these values are now 0.63 and 16, respectively. The higher sensitivities reflect the higher degree of nonlinearity within the option pricing formula as sovereign assets move closer to the distress barrier. This is indicative of observed nonlinear value changes in actual credit events.

A similar procedure can be applied to illustrate the opposite effects of capital inflows. Sustained capital inflows typically result in some exchange rate appreciation, improvement in domestic debt prices, and lower financial market volatility. Capital inflows may also provide space for an increase in international reserves which may necessitate sterilization operations. Based on this scenario, the value of sovereign assets is assumed to rise US$195 billion while its volatility drops to 37 percent. Also, international reserves is assumed to rise by US$5 billion and the increase in the dollar value of domestic currency liabilities is a reflection of both sterilization and exchange rate appreciation. The right column of Table 6 (Scenario 2) displays the contingent claim sovereign balance sheet, credit risk indicators, and sensitivities after capital inflows. The increase in sovereign asset value and reduction in volatility yield the expected decrease in credit risk and sensitivity relative to the baseline. Distance to distress rises above two standard deviations and risk-neutral probability of default decreases by half to 4 percent. Risk-neutral spreads on foreign currency debt decline as the value of risky foreign currency debt approaches its default free value. Each of the sensitivity measure decreases relative to the baseline from the improved sovereign asset value and volatility with respect to the distress barrier.

C. Monte Carlo Simulations

The scenario analysis above yields three related point estimates for the credit risk indicators, one from the baseline calibration and two from a negative and positive shock. While such scenario analysis may be useful in examining a specific event, it only reveals a very small view of the possible set of market disturbances. Scenario analysis to recreate a specific event is always subject to the criticism that market stress scenarios, in fact, rarely repeat themselves. Monte Carlo simulation methods can be used to systematically deal with multiple scenarios, yielding probability distributions for risk indicators and value-at-risk

(VaR) measures.[32] The Monte Carlo procedure implemented in this section takes random draws from hypothetical forward distributions for domestic interest rates and the exchange rate. Following the process outlined in Box 2, Monte Carlo simulations are conducted on the baseline sovereign balance sheet presented in the previous section.

Probability distributions for distance to distress, risk-neutral default probability, risk-neutral spreads, and the value of sovereign assets resulting from the simulation are reported in Figure 7. While the mean distance to distress remains 1.4 standard deviations, the same value as reported in Table 4, the distribution reveals a confidence interval for distance to distress based on the sample exchange rate and interest rate distributions. For example, from Figure 7, the lower 5 percent probability for distance to distress is 0.9 standard deviations, the upper 5 percent probability for risk neutral default probability is 18 percent, and the upper 5 percent probability for risk neutral spreads is 387 basis points. In other words, given the assumed exchange rate and interest rate distributions and correlation, distance to distress remains above 0.9 standard deviations, risk neutral default probability remains below 18 percent, and risk neutral spreads remain below 387 basis points 95 percent of the time. Finally, the 5 percent lower bound on sovereign assets is US$160 billion, making the implied sovereign asset VaR equal to US$15 billion.

VaR measures are often used to evaluate both market and credit risk in the financial sector.[33] In the financial sector, VaR typically defines a level of capital which, for a high degree of confidence, is an upper bound on the amount of gains or losses to a portfolio from market or credit risk. On the sovereign balance sheet, VaR by corollary, could be defined as the upper bound on the amount of gains or losses to implied sovereign asset value from market risk.[34] Just as a bank or asset manager is required to hold capital in reserve to protect against market

[32] While Monte Carlo simulations are able to handle many thousands of possible events, they produce a random set of outcomes based on the market characteristics assumed, which may or may not predict potential shocks. The simulation process will only produce as many extreme events as dictated by the distribution assumption of the market variables. To be comprehensive, simulation procedures should be combined with various scenario assumptions to produce a set of stress outcomes.

[33] See Jorian (2000). VaR models estimate the exposure of a portfolio, or the equivalent set of positions, to market risk. The measure captures the expected maximum loss and is usually expressed within a confidence interval.

[34] Two other sovereign VaR measures can be calculated. The first, sovereign capital-at-risk, is an extension of sovereign VaR for the central bank. The probability distribution of the residual value of "capital" or junior claim of the monetary authorities is calculated and a confidence level attached to the risk that the monetary authorities cannot meet its commitments. Blejer and Schumacher (1999) use a similar construction. The second, sovereign credit-at-risk, is the upper bound on gains or losses due to credit risk, which in this case is the value of the guarantee to the banking system. See Gapen, Gray, Lim, and Xiao (2004) for an example of how this could be modeled.

or credit loss, governments often identify a need to acquire sufficient levels of foreign currency reserves or insurance arrangements to protect against adverse market developments. The sovereign value-at-risk measure can be used as a tool to gauge whether the level of reserves is sufficient to protect against the risk of "sudden stops," or to maintain debt sustainability against adverse economic shocks.

D. Evaluating Policy Design

Using the Monte Carlo baseline simulation as a starting point, potential policy choices can be evaluated. For example, changes in the level of reserves, alternative debt structures, or the use of risk mitigation instruments like insurance contracts can be tested. The new policy option modifies the sovereign balance sheet and simulations using draws from the same interest rate and exchange rate distributions will reveal new distributions of risk indicators which can be evaluated against the original baseline configuration.[35] The example of debt management with alternative debt structures is examined first followed by a strategy for reserve accumulation. Finally, a combination of debt and reserve management is considered.

- **Debt management.** Panel 1in Figure 8 illustrates an example of liability management, whereby US$10 billion in foreign currency debt is replaced with an equal amount of interest rate linked domestic currency debt to examine the impact of reduced exchange rate exposure. As a result, the distress barrier falls to US$90 billion while domestic currency liabilities increase by US$10 billion. The new Monte Carlo simulations on this adjusted balance sheet yield improvements in the risk indicators. The mean values and confidence intervals for distance to distress, risk-neutral default probability, and mean risk-neutral spreads all improve.

- **Reserve management**. Panel 2 of Figure 8 illustrates the example of reserve accumulation financed with an equal amount of domestic currency debt such that the level of sovereign assets and interest rate linked domestic currency liabilities both increase by US$10 billion. This scenario could be viewed as a proactive strategy to accumulate reserves or reflect capital inflows, and therefore, the increase in domestic currency debt is the result of sterilization. The operation yields improvements in the risk indicators—as seen in the mean values and lower 5 percent probability distributions—although the margin of improvement is less than that found in the example on debt management.

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[35] Simulating the adjusted sovereign balance sheet under the same exchange rate and interest rate distributions and correlation is subject to the Lucas critique; namely that these distributions and correlations are derived from market expectations which are likely to change with the shift in policy. In this debt management example, the expected future exchange rate could be more appreciated with lower volatility given the lower levels of foreign currency debt. Consequently, the simulations conducted in this paper should only be viewed as illustrative of potential impacts from policy changes. See Best (2000) for additional discussion on the limits of stress testing.

- **Asset and liability management**. The reserve and debt management strategies above can also be implemented simultaneously, as shown in Panel 3 of Figure 8. The distress barrier declines to US$90 billion, the amount of domestic currency debt increases by US$20 billion, and the level of foreign currency reserves increases by US$10 billion. The effect of simultaneously enacting both strategies yields improvements in the risk indicators by an amount equal to more than the sum of the two strategies individually, reflecting model nonlinearity. Combining the two strategies is advantageous since the debt management operation reduces the distress barrier while the reserve accumulation strategy leaves the mean value of sovereign assets nearly unchanged relative to the baseline.

Deciding on the efficacy of any of the above strategies involves a systematic weighting of the trade-offs inherent in each case. There are clear elements in each of the three alternative strategies that are beneficial from a policy perspective (the reduction in exchange rate exposure and increase in reserves) which need to be balanced against the clear negatives from a balance sheet perspective (the increase in domestic currency interest bearing obligations). The contingent claims balance sheet risk indicators can therefore be useful in guiding policy design given its ability to compare different policy options using quantifiable risk indicators.

VII. Next Steps

A. A Robust Framework for Reserve Management

The application of contingent claims analysis and sovereign VaR to reserve management is a stark departure from accounting indicators commonly used for reserve management. One widely used indicator of reserve adequacy is the ratio of foreign currency reserves to total public and private short-term foreign currency debt. Both public and private sector debt is included since reserves of the public sector must facilitate transactions related to economy-wide financing requirements. However, the simple accounting ratio of reserves to total short-term foreign currency debt is deficient when it comes to risk analysis because it does not take uncertainty of balance sheet risks into account. Applying a broad-based rule for an appropriate ratio of reserve coverage uniformly across countries implicitly assumes all sovereign balance sheet risks are similar and neglects cross-country differences in sovereign balance sheet risk.[36]

In contrast, an appropriate target for reserve adequacy could be based on a level of reserves that minimizes instances of distress using the contingent claims risk indicators. For example, an adequate level of reserves could be defined as the level of reserves that keeps distance to distress above a desired standard deviation 95 percent of the time based on the likely

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[36] IMF (2000) examines three ratios: reserves to imports, reserves to monetary aggregates, and reserves to public and private short-term foreign currency debt by residual maturity. The report concludes that reserves to short-term foreign currency debt is a superior measure and recommends a ratio of 1 be a lower bound for adequate reserve coverage.

exchange rate process. Adequate reserve coverage could also target a basket of credit risk indicators by setting reserve levels to maintain the combined set of indicators at target levels for a specific confidence interval. In sum, reserve management using this framework examines the impact of the level and volatility of reserves as a component of the wider sovereign asset value and volatility with a link to the balance sheet risk indicators. The application of contingent claims in analyzing sovereign credit risk can be adapted to include many different aspects of reserve management, including the currency composition of reserves, or various other risk mitigation techniques.[37]

B. A Robust Framework for Debt Sustainability

In addition to providing a framework for reserve management, the use of contingent claims to analyze sovereign risk is well-suited for robust debt sustainability analysis. Traditional debt sustainability analysis has focused on ratios of current and forecasted debt-to-GDP as the primary criterion for deciding whether the public sector debt remains on a sustainable path, usually without explicitly incorporating uncertainty in a systematic, coherent framework. The following elements indicate why the approach in this paper could provide the basis for a more robust framework for debt sustainability analysis:

- The contingent claims sovereign balance sheet translates balance sheet risks into quantifiable risk indicators. In this framework, debt sustainability could be defined as the debt structure which keeps key credit risk indicators below (or above) certain threshold levels for a given confidence level. In contrast, the debt-to-GDP ratio identifies an element of sovereign risk, but is not part of a structural framework that measurably relates debt payments with the capacity to pay. For example, the contingent claims structural framework is able to assess the impact of changes in the level of reserves on sovereign risk whereas the debt-to-GDP ratio remains invariant to such changes.

- The quantitative sovereign credit risk indicators described in this paper incorporate uncertainty and volatility. Higher market uncertainty is often translated into higher interest rate and exchange rate volatility, widening the forward distributions on both variables and increasing the volatility of sovereign assets. Distance to distress will fall, probability of default will rise along with spreads on foreign currency debt, and the expected loss on risky foreign currency debt will increase. However, the debt-to-GDP ratio does not change with an increase in sovereign asset volatility and would therefore miss an important component of risk analysis

[37] Caballero and Panageas (2005) examine various instruments and risk mitigation strategies that policymakers could implement in addition to traditional reserve accumulation in a model of sudden stops in capital flows.

- The contingent claims sovereign balance sheet includes an assessment of maturity or rollover risk through construction of the distress barrier.[38] The debt-to-GDP ratio does not change if a decrease in short-term foreign currency debt is matched by an equal book value increase in long-term foreign currency debt. The use of the contingent claims sovereign balance sheet reflects this change by signaling a decrease in sovereign risk due to the more favorable debt profile.

- Finally, the contingent claims approach incorporates nonlinear value changes. The use of nonlinear modeling in a structural framework captures complex relationships and more accurately conveys the nonlinear nature of credit events. During periods of stress, small changes in interest rates, exchange rates, and/or volatility can result in large changes in sovereign risk on the margin. An accounting ratio like debt-to-GDP is not capable of this level of complexity, nor is it released with enough frequency to enable its use during periods of stress where vulnerabilities may build or subside rapidly.

Using contingent claims to model sovereign credit risk therefore offers several advantages over the traditional debt-to-GDP analysis. Additional research in this direction could prove useful and would require extension of the framework to a medium-term setting while incorporating the outlook for relevant economic and policy variables.

VIII. Conclusions

This paper develops a comprehensive new framework to measure and analyze sovereign risk by applying the contingent claims approach to the balance sheet of the combined government and monetary authorities. A marked-to-market balance sheet is constructed that provides a structural framework that identifies balance sheet risks, incorporates uncertainty, and yields quantifiable risk indicators. The main outputs of this framework include sovereign credit risk indicators, sensitivity measures, and sovereign value-at-risk. These sovereign risk indicators incorporate both forward-looking market prices and nonlinear changes in values, and should consequently have greater predictive power in estimating sovereign credit risk than would traditional macroeconomic vulnerability indicators or accounting based ratios.

Application to 12 emerging market economies show the risk indicators to be robust and significant when compared to market observed credit spreads on foreign currency debt, even

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[38] This is true whether one uses the simplified distress barrier in this paper (short-term foreign currency debt and interest plus one-half long-term foreign currency debt) or a more sophisticated approach (short-term foreign currency debt and interest plus the present discounted value of long-term foreign currency debt and interest). Both approaches would reflect an increase in sovereign risk if long-term foreign currency debt was traded for equal book value amounts of short-term foreign currency debt. The distress barrier under the second approach, however, would be more sensitive to near-term repayment humps that would carry a higher weight in the distress barrier than a similar payment profile further out the maturity scale.

though the spreads were not used as inputs. This lends support for the approach, as well as illustrates that the level and variation of forward exchange rates and other market variables contain valuable information for analyzing sovereign credit risk.

Using the contingent claims approach to analyze sovereign risk has several merits from a policy perspective. The ability of the contingent claims approach to provide a structural interpretation of the sovereign balance sheet, unique to each economy, is a valuable contribution in the area of policy design and risk management, translating policy choices and changing economic conditions directly into quantitative indicators of financial soundness. This paper describes how the framework can be used with scenario and simulation analysis to evaluate shocks and the impact of corrective policies. Policymakers can observe how negative economic shocks worsen sovereign financial soundness through capital outflows, depreciating the exchange rate, or slower economic growth. Policymakers can use this tool to design and implement risk-mitigation strategies to reduce sovereign balance sheet risk. Equally important, the set of tools available to policymakers have direct links to the assets and liabilities in the sovereign balance sheet and can influence market expectations. In response to changing economic conditions, active policy decisions to alter the primary fiscal surplus, level of interest rates, structure of debt, or reserve intervention policy can mitigate or offset shocks to the government balance sheet. The ability of the contingent claims balance sheet risk to measurably assess the potential policy mix is an important element of strategic planning, and offers policymakers the valuable opportunity to rank policy options.

The contingent claims framework can be adapted and extended in several important directions. It is well-suited for a more robust analysis of debt sustainability as compared with the widely used debt-to-GDP ratio which is a static, backward-looking indicator. The framework can also be used to estimate an appropriate target for reserve adequacy, where adequacy could be defined as the level of reserves that minimizes the probability of distress.

Box 1. From Corporate to Sovereign Risk Analysis

It is useful to point out the key similarities between the sovereign balance sheet and balance sheets of individual firms. On the asset side,

- **Firms** have main assets consisting of cash and the present value of future earnings (stream of revenues minus expenditures). The firm has other assets including property, plant, equipment, and inventory. The firm may have also contingent assets from a parent company, or implicit guarantees to subsidiaries.

- The **sovereign** has, on its consolidated balance sheet, international reserves and present value of the net fiscal surplus (stream of revenues minus expenditures). Sovereign assets may increase due to contingent financing arrangements with multilateral or other sources, or may be reduced by the cost of implicit or explicit guarantees to financial institutions or other too-important-to fail entities. The sovereign has land and other assets, which are unlikely to be sold, so they do not enter into the expected government revenue stream and thus are not included in this definition of asset.

On the liability side, firm liabilities may include senior debt, subordinated debt, and equity. Market capitalization of the firm is equal to price of equity multiplied by the number of shares issued. Sovereign liabilities include foreign currency debt. The sovereign also has local currency debt and base money, which when multiplied by the exchange rate yields the foreign currency value of domestic currency liabilities.

Default risk in the corporate and sovereign sectors are also similar from a valuation perspective:

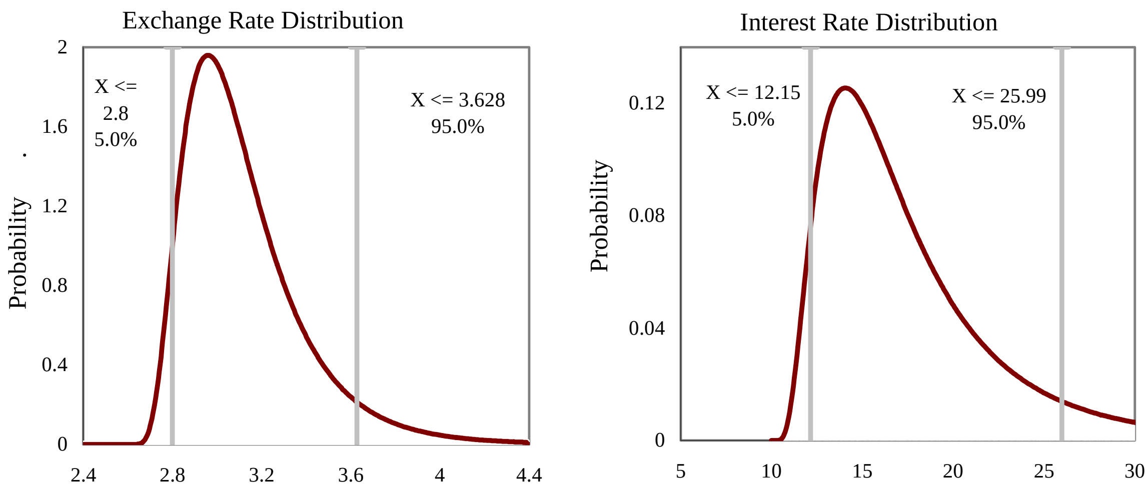
- **Corporate default.** Corporate sector defaults trigger a bankruptcy process, well-defined in some countries and less well-defined in others, whereby creditors are assigned their claim to firm assets based on the legally defined seniority of liabilities in the capital structure. Since debt is senior to equity in the firm liability structure, bondholders have senior legal claims to remaining firm assets in the event of default. A review of corporate sector defaults reveals that senior bondholders exercise their control in various ways post default. In some cases, bondholders choose to sell liquidate remaining assets to obtain cash payment. In other cases, bondholders choose to replace management while receiving new claims, oftentimes in the form of equity.

- **Sovereign default.** While sovereign defaults do not trigger a well-defined bankruptcy process that applies equally across countries, instances of sovereign default trigger a restructuring process whereby predefault liabilities are exchanged for postdefault liabilities. In this restructuring process, holders of sovereign liabilities do not receive similar legal claims to ownership of sovereign assets in the event of a sovereign default (e.g., bondholders cannot assume control of the policy apparatus, possess public sector entities, or liquidate assets). Instead, the holders of sovereign debt have a claim on restructured debt of lower value in the event of default.

From a valuation perspective, default risk in a sovereign setting is similar to default risk in the corporate sector. The present value of the expected loss in the sovereign setting is associated with receiving restructured debt of lower value after default while in the corporate setting it is associated with post-default cash payouts or new claims at lower value. Bondholders in both cases value their claims at their default-free value minus the present value of expected loss, which can be estimated using an implicit put option.

Box 2. Implementing the Monte Carlo Simulation

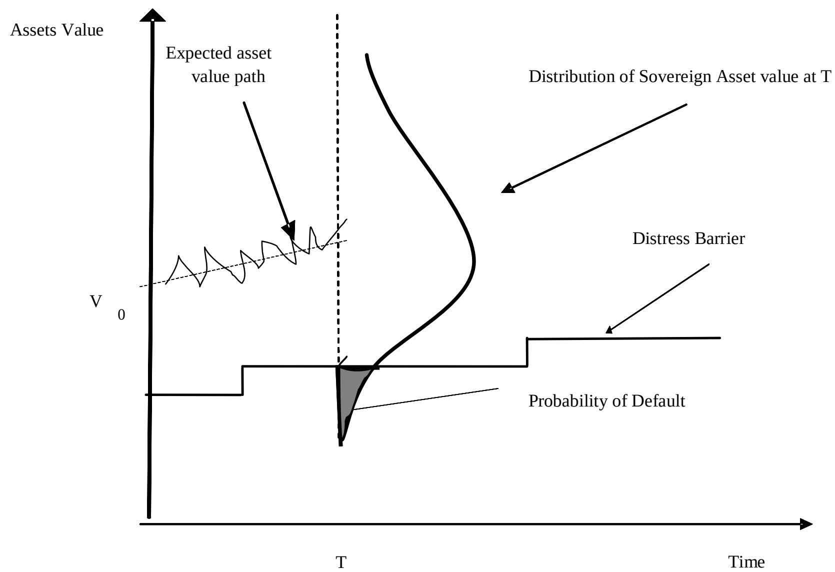
The Monte Carlo simulation procedure applied in this section takes random draws from hypothetical forward distributions for both domestic interest rates and exchange rates and calculates the effects of these variables on balance sheet values and risk indicators. The one-year forward exchange rate is assumed to be 3 units of the domestic currency to 1 U.S. dollar and the one-year forward interest rate is assumed to be 17 percent. Lognormal distributions for each were constructed based on recently observed market patterns in several emerging market economies, as shown in the figures below. The correlation between exchange rates and interest rates was set at 0.6, meaning that the Monte Carlo simulation conducts sample draws such that exchange rate depreciations are generally associated with higher interest rates and vice versa.



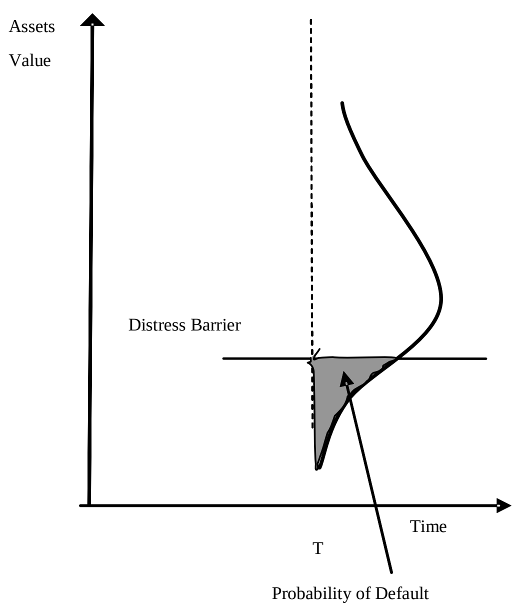
The Monte Carlo simulation procedure then selects random draws from these hypothetical distributions. The sample forward exchange rate is applied to the contingent claims sovereign balance sheet in the translation of domestic currency assets and liabilities into their respective U.S. dollar values. In contrast to exchange rate variations, simulating the effect of interest rate changes requires additional assumptions. Broad money is assumed to comprise half of domestic currency liabilities with the remainder in interest rate-linked domestic debt. The interest rate draw is applied to the existing domestic currency debt for a period of three years and then is assumed to return to 17 percent. If the realization of interest rates in the random draw is above the assumed 17 percent forward interest rate, the discounted marginal increase in interest costs are subtracted from the value of sovereign assets to reflect higher debt service costs. Alternatively, if the interest rate draw is below the assumed forward interest rate, then this discounted decrease in debt service costs is added to the value of sovereign assets.

The resulting sovereign balance sheet values from each random draw are then used to compute the new set of risk indicators. In contrast to the point estimates for the balance sheet risk indicators that result from scenario analysis, the process of conducting random samples from distributions of exchange rates and interest rates results in probability distributions for the relevant risk indicators.

Figure 1. Distribution of Sovereign Asset Value and the Distress Barrier



Lower Sovereign Asset Value



Lower Sovereign Asset Value and Higher Volatility

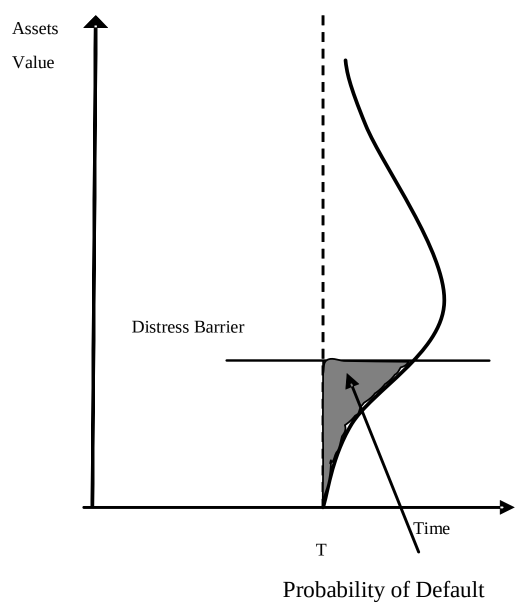


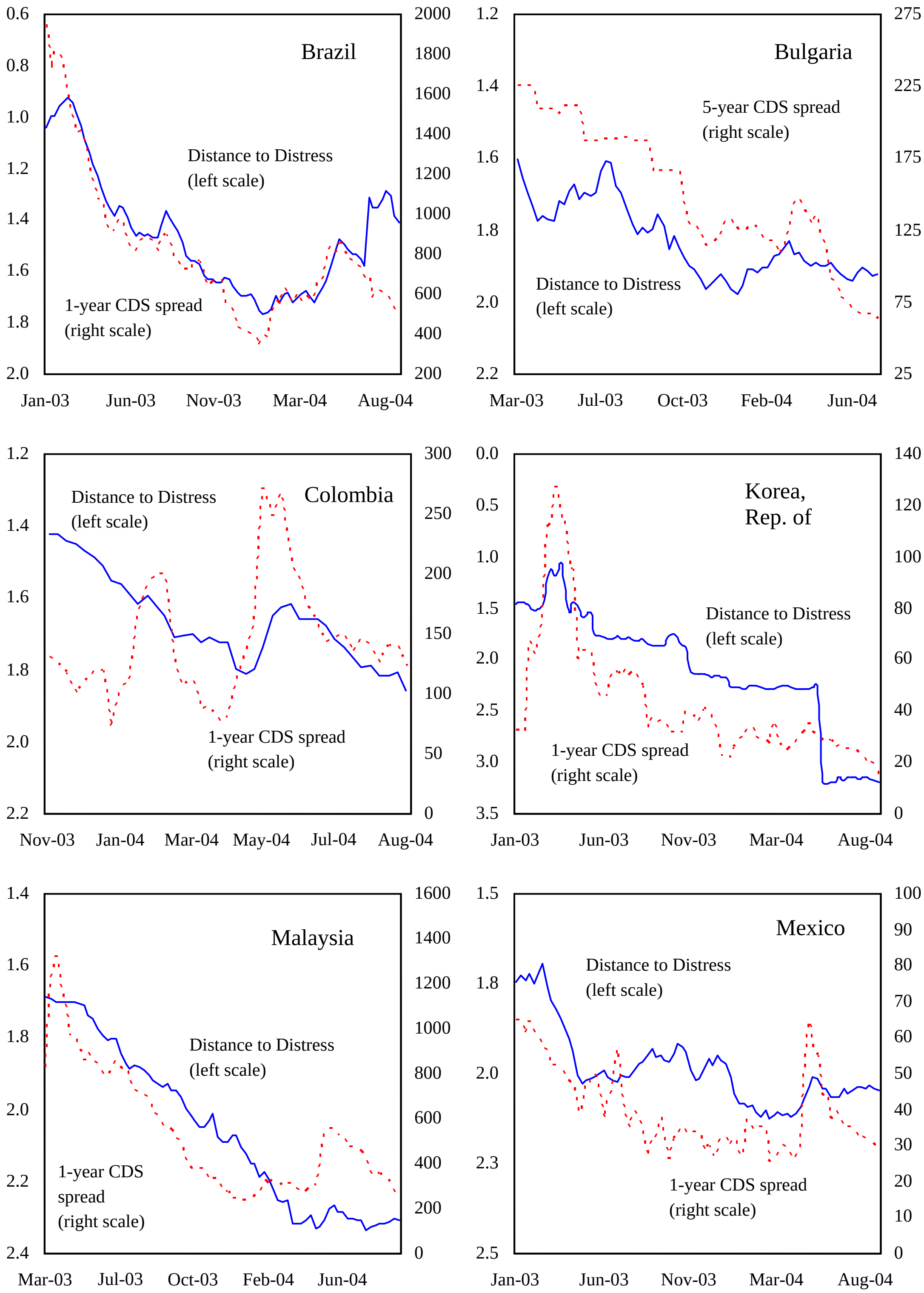
Figure 2. Segregated Balance Sheets of the Government and Monetary Authority

GOVT ASSETS	**GOVT LIABILITIES**
Claim on a Portion of International Reserves (of Monetary Authority) Other Public Sector Assets	*Credit to Government (from Monetary Authority)* Domestic Currency Debt Foreign Currency Debt Implicit and Explicit "Too-important-to-fail" Guarantees
MONETARY AUTHORITY ASSETS	**MONETARY AUTHORITY LIABILITIES**
International Reserves *Credit to Government*	Base Money *Government Claim on Portion of International Reserves*

Figure 3. The Consolidated Contingent Claims Public Sector Balance Sheet

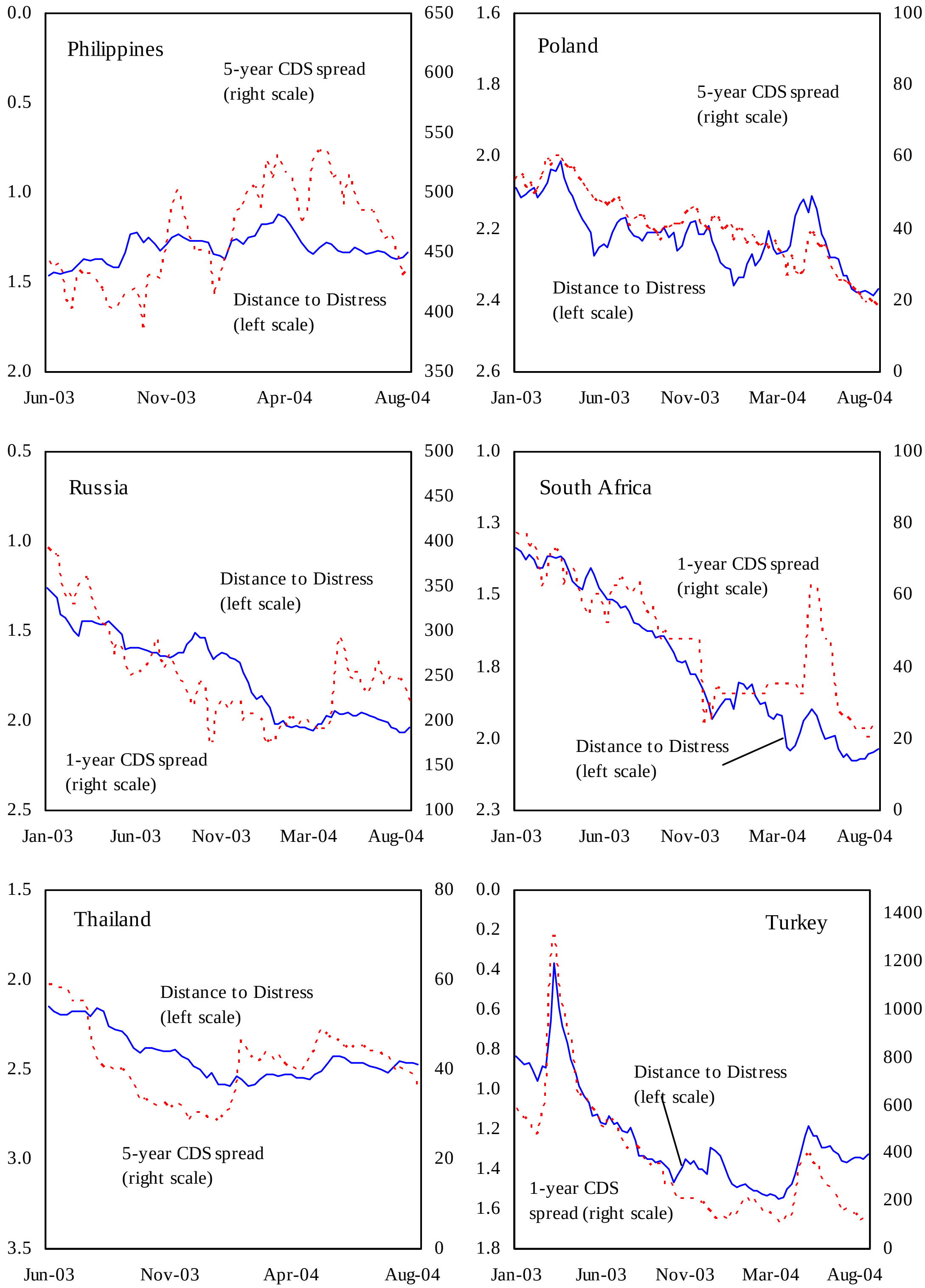
ASSETS	**LIABILITIES**
International Reserves **Domestic Currency Assets, in Foreign Currency Terms** [= Other Assets – Guarantees]	**Value of Domestic Currency Liabilities, in Foreign Currency Terms** [= Domestic Currency Debt + Base Money] **Foreign Currency Debt**

Figure 4. Distance to Distress and Credit Default Swaps



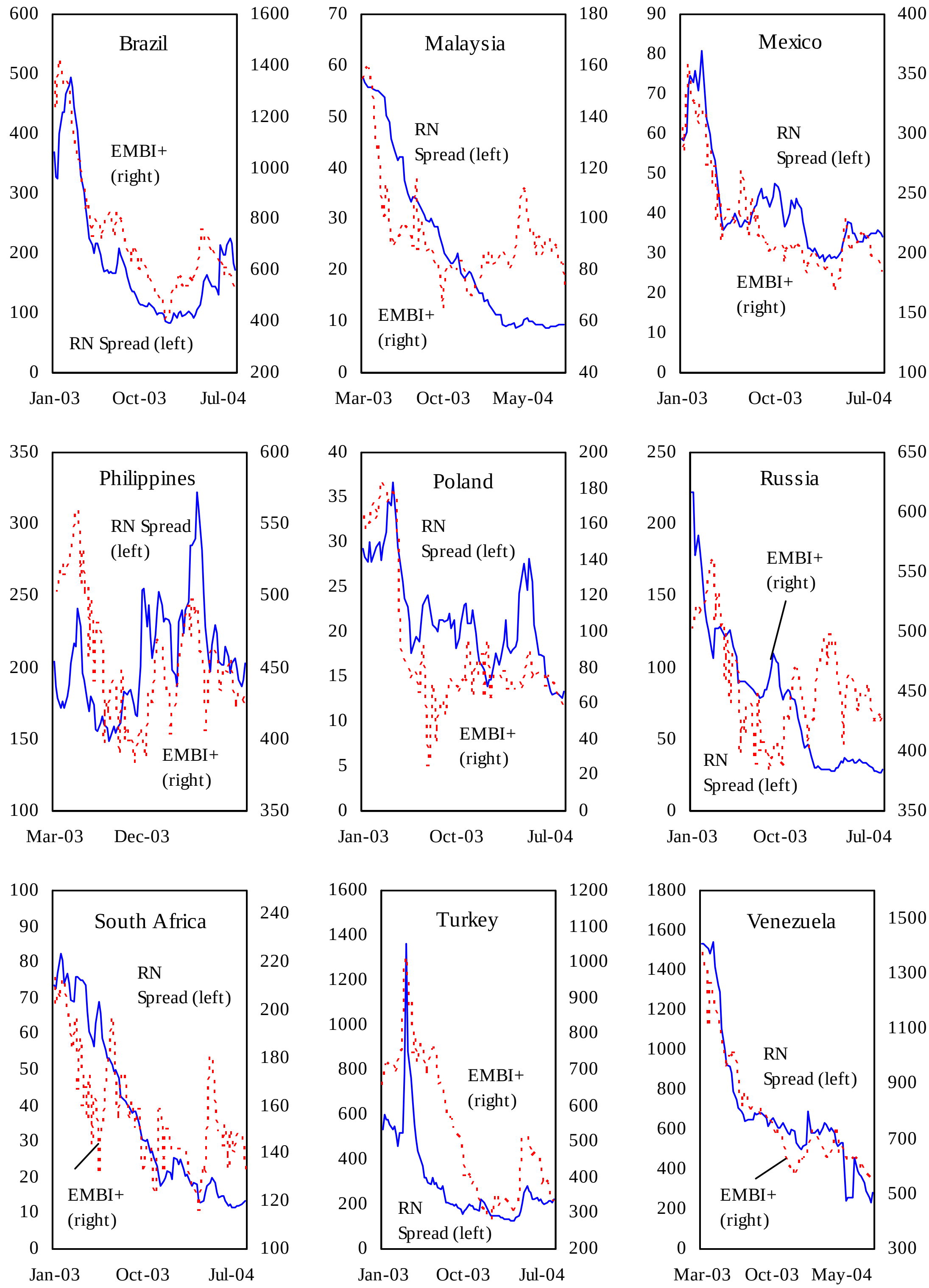
Source: Moody's MfRisk, Bloomberg.

Figure 4, contd. Distance to Distress and Credit Default Swaps



Source: Moody's MfRisk, Bloomberg.

Figure 5. Model Credit Spread and J.P. Morgan EMBI+ Country Index



Source: Moody's MfRisk, Bloomberg.

Figure 6. Market Observed Spreads and Model Distance to Distress

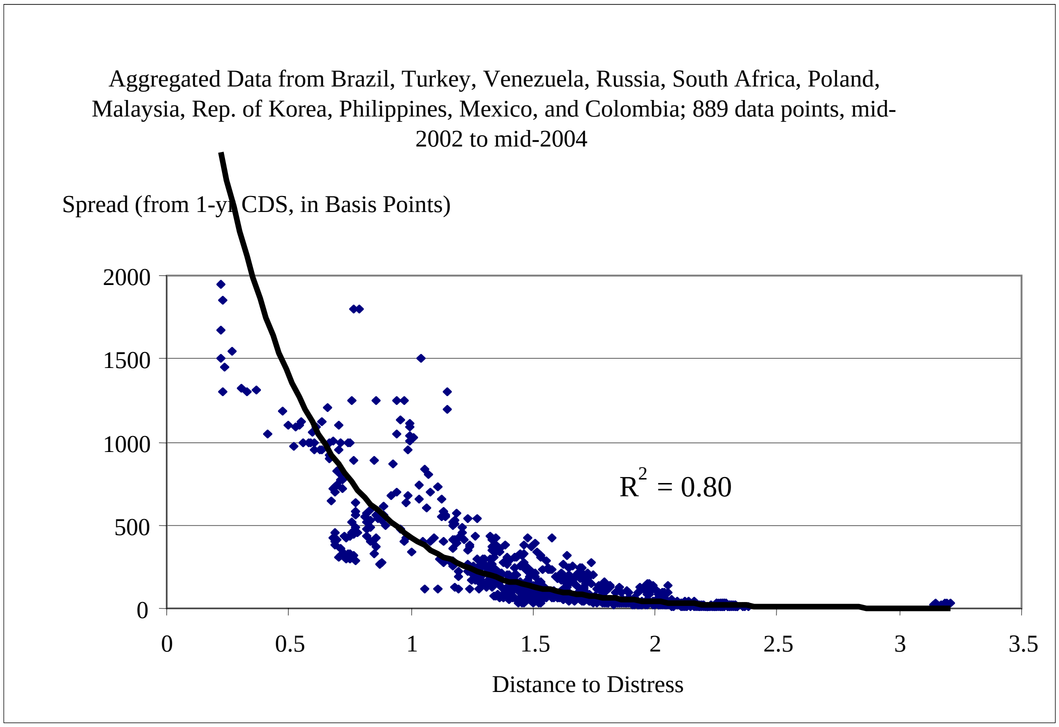


Figure 7. Monte Carlo Simulations: Hypothetical Sovereign

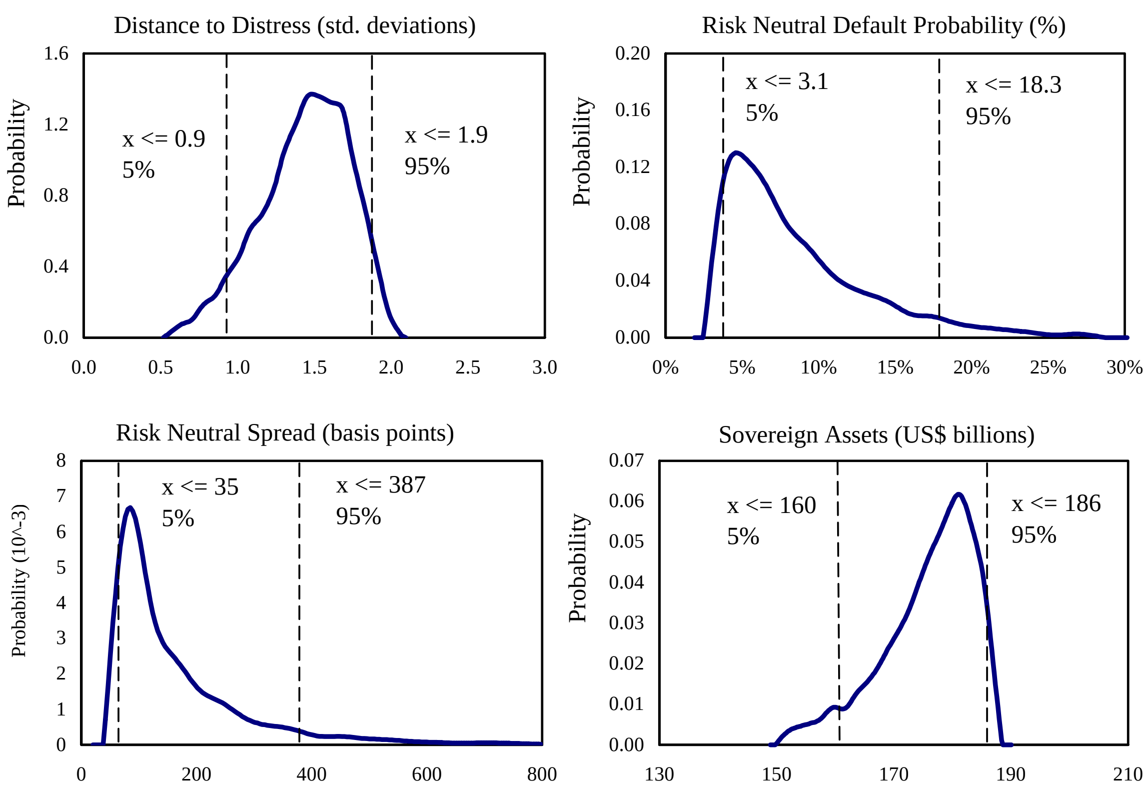
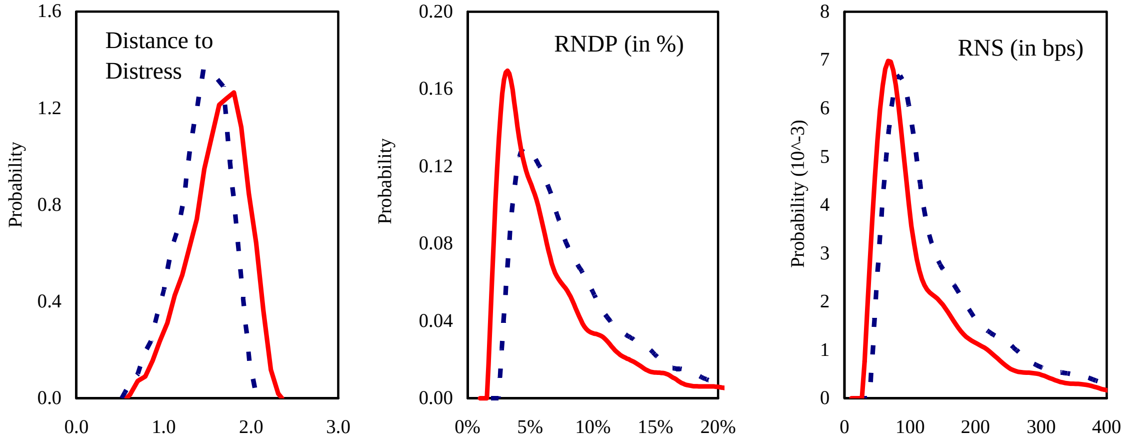
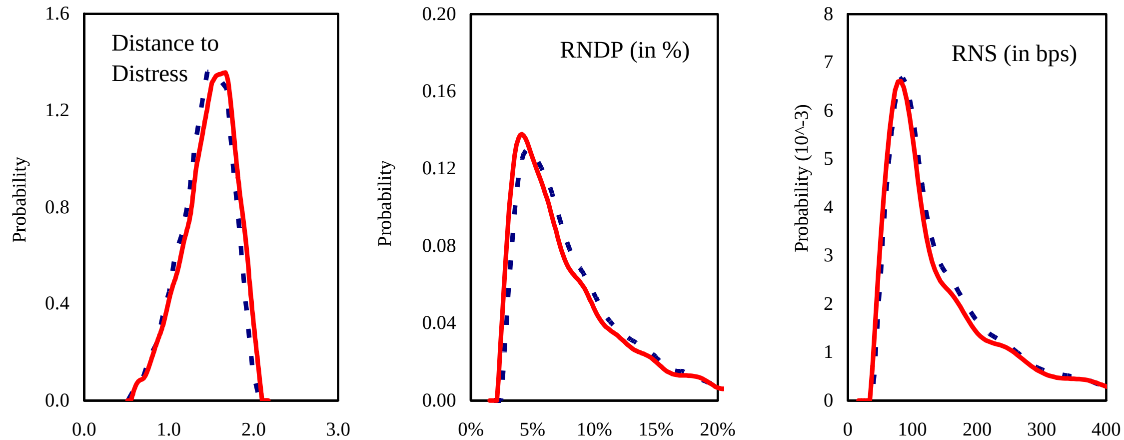


Figure 8. Evaluation of Policy Options

Panel 1. Debt Management Risk Indicators (solid) Versus Baseline Risk Indicators (dashed).



Panel 2. Reserve Management Risk Indicators (solid) Versus Baseline Risk Indicators (dashed).



Panel 3. Combined Debt and Reserve Management Risk Indicators (solid) vs. Baseline Risk Indicators (dashed).

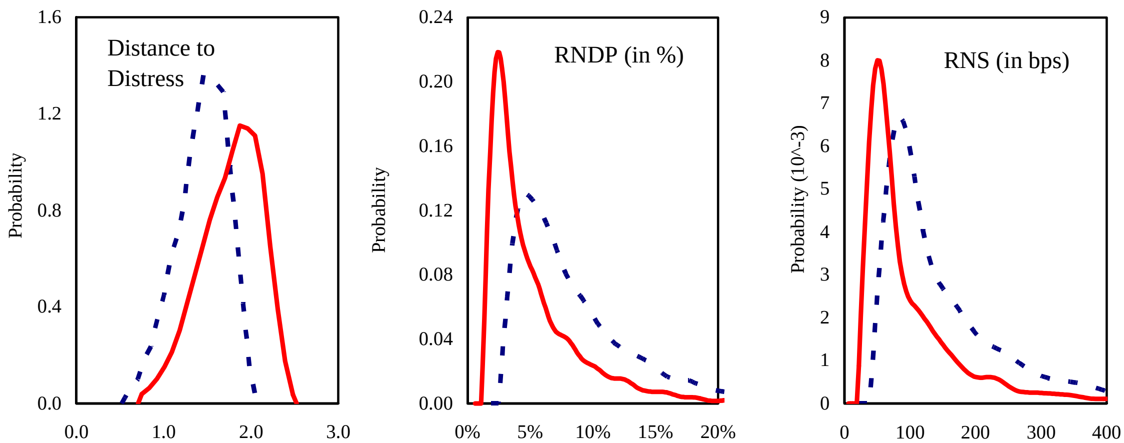


Table 1. Spearman Rank Correlation: Sovereign Risk Indicators and Actual Data

	Distance to Distress and:				Risk Neutral Spread and:			
Country	1-year CDS Spread	3-year CDS Spread	5-year CDS Spread	Country EMBI+ Spread	1-year CDS Spread	3-year CDS Spread	5-year CDS Spread	Country EMBI+ Spread
Brazil	-0.68**	-0.79**	-0.80**	-0.81**	0.70**	0.82**	0.82**	0.83**
Bulgaria	n/a	-0.72**	-0.91**	n/a	n/a	0.72**	0.83**	n/a
Korea, Rep of	-0.83**	-0.85**	-0.88**	n/a	0.84**	0.85**	0.89**	n/a
Malaysia	-0.72**	-0.73**	-0.14	-0.36**	0.72**	0.73**	0.15	0.39**
Mexico	-0.44**	-0.62**	-0.73**	-0.72**	0.44**	0.62**	0.73**	0.73**
Philippines	-0.33*	-0.43**	-0.53**	-0.20	0.33*	0.43**	0.54**	0.17
Poland	-0.16	-0.68**	-0.69**	-0.44**	0.06	0.67**	0.69**	0.45**
Russia	-0.29**	-0.54**	-0.66**	-0.47**	0.30**	0.54**	0.67**	0.47**
South Africa	-0.80**	-0.76**	-0.75**	-0.47**	0.86**	0.77**	0.75**	0.64**
Thailand	-0.29	n/a	-0.28*	n/a	0.41*	n/a	0.27*	n/a
Turkey	-0.83**	-0.84**	-0.84**	-0.85**	0.82**	0.83**	0.83**	0.85**
Venezuela	-0.29*	-0.22	-0.20	-0.89**	0.33*	0.27	0.22	0.90**

** Denotes significance at 1 percent level.

* Denotes significance at 5 percent level.

Table 2. Regression Output: Risk-Neutral Spreads and CDS Spreads

$$\ln\left(\boldsymbol{CDS}_i\right) = \boldsymbol{i}\alpha_i + \boldsymbol{\beta}\ln\left(\boldsymbol{RNS}_i\right) + \boldsymbol{\varepsilon}_i$$

R-squared	0.88
Adjusted R-squared	0.88
Log likelihood	-645.02
F-statistic	640.78
Prob(F-statistic)	0.00

Indep. Variables / Country	Constant	Coefficient	Std. Error	t-Statistic	Prob.
ln(RNS)		0.52	0.02	24.53	0.00
Brazil	3.43		0.12	28.09	0.00
Colombia	2.54		0.11	22.34	0.00
Korea, Rep. of	2.59		0.07	38.50	0.00
Malaysia	1.54		0.08	18.57	0.00
Mexico	1.72		0.09	18.42	0.00
Philippines	2.53		0.12	20.50	0.00
Poland	1.20		0.08	14.55	0.00
Russia	2.72		0.10	26.94	0.00
South Africa	2.09		0.09	23.63	0.00
Turkey	2.98		0.13	23.04	0.00
Venezuela	2.94		0.15	20.14	0.00

Table 3. Regression Output: Risk-Neutral Spreads and EMBI+ Spreads

$$\ln\left(\boldsymbol{EMBI}_i\right) = \boldsymbol{i}\alpha_i + \boldsymbol{\beta}\ln\left(\boldsymbol{RNS}_i\right) + \boldsymbol{\varepsilon}_i$$

R-squared	0.96
Adjusted R-squared	0.96
Log likelihood	340.48
F-statistic	1548.00
Prob(F-statistic)	0.00

Indep. Variables / Country	Constant	Coefficient	Std. Error	t-Statistic	Prob.
ln(RNS)		0.15	0.02	6.30	0.00
Brazil	5.69		0.11	50.86	0.00
Colombia	4.68		0.08	60.98	0.00
Korea, Rep. of	5.41		0.11	48.94	0.00
Malaysia	4.04		0.07	60.30	0.00
Mexico	4.78		0.08	58.74	0.00
Philippines	5.29		0.12	42.88	0.00
Poland	3.79		0.08	47.83	0.00
Russia	5.05		0.09	54.28	0.00
South Africa	4.52		0.07	61.41	0.00
Turkey	5.26		0.12	43.85	0.00
Venezuela	5.61		0.14	39.41	0.00

Table 4. Regression Output: Default Probability

$$\ln\left(\boldsymbol{MIDP}_i\right) = \boldsymbol{i}\alpha_i + \boldsymbol{\beta}\ln\left(\boldsymbol{RNDP}_i\right) + \boldsymbol{\varepsilon}_i$$

R-squared	0.93
Adjusted R-squared	0.93
Log likelihood	-157.18
F-statistic	770.00
Prob(F-statistic)	0.00

Indep. Variables / Country	Constant	Coefficient	Std. Error	t-Statistic	Prob.
ln(RNDP)		0.23	0.03	7.38	0.00
Brazil	0.61		0.08	7.94	0.00
Colombia	0.24		0.07	3.37	0.00
Korea, Rep. of	-0.77		0.04	-21.63	0.00
Malaysia	-1.44		0.03	-49.84	0.00
Mexico	-0.90		0.04	-25.61	0.00
Philippines	0.50		0.09	5.52	0.00
Poland	-1.65		0.04	-38.67	0.00
Russia	0.16		0.04	3.82	0.00
South Africa	-0.79		0.05	-17.13	0.00
Turkey	0.64		0.08	8.39	0.00
Venezuela	1.08		0.08	12.76	0.00

Table 5. Example Contingent Claims Sovereign Balance Sheet Risk Indicators

Contingent Claim Sovereign Balance Sheet	(US$ billion, unless indicated)
Value of sovereign assets (implied)	175
Foreign reserves (observed value)	40
Sovereign asset less reserves (implied)	135
Value of risky foreign currency debt	94.5
Distress barrier 1/	100
PV of distress barrier 1/	96
PV of expected losses (= implict put option)	1.5
Value of local currency liabilities 1/	80.5
Volatility of asset (implied)	38%
Credit Risk Indicators	
Distance to distress 2/	1.4
Risk-neutral default probability (RNDP)	8%
Risk-neutral spread (RNS) 3/	115
Sensitivity Measures 4/	
Change in distance to distress / 1% change in assets 2/	-0.03
Change in distance to distress / 1% change in asset vol. 2/	-0.05
Change in RNDP / 1% change in assets	0.41
Change in RNS / 1% change in assets 3/	7
Change in RNS / 1% change in asset vol. 3/	16
Change in PV expected loss / 1 % change in assets	0.07
Change in PV expected loss / 1 % change in asset vol.	0.15

1/ Model inputs. Remainder are model outputs.
2/ In standard deviation of sovereign asset value.
3/ Spread in basis points.
4/ Based on a 1 percent change in sovereign asset value (e.g. from 175 to 176.75) and sovereign asset volatility (e.g. from 38 percent to 39 percent).

Table 6. Alternative Scenarios and Contingent Claim Sovereign Balance Sheet Risk Indicators

	Scenario 1	Baseline	Scenario 2
Contingent Claim Sovereign Balance Sheet	(US$ billion, unless indicated)		
Value of sovereign assets	155	175	195
Foreign reserves 1/	35	40	45
Sovereign asset less reserves	120	135	150
Value of risky foreign currency debt	93	94.5	96
Distress barrier 1/	100	100	100
PV of distress barrier 1/	96	96	96
PV of expected losses (= implict put option)	3	1.5	0.5
Value of local currency liabilities 1/	62	80.5	99
Volatility of asset	43%	38%	37%
Credit Risk Indicators			
Distance to distress 2/	1.0	1.4	2.1
Risk-neutral default probability (RNDP)	17%	8%	4%
Risk-neutral spread (RNS) 3/	325	115	60
Sensitivity Measures 4/			
Change in distance to distress / 1% decrease in assets 2/	-0.02	-0.03	-0.03
Change in distance to distress / 1% increase in asset vol. 2/	-0.03	-0.05	-0.06
Change in RNDP / 1% decrease in assets	0.63	0.41	0.23
Change in RNS / 1% decrease in assets 3/	16	7	3
Change in RNS / 1% increase in asset vol. 3/	28	16	9
Change in PV expected loss / 1 % decrease in assets	0.15	0.07	0.03
Change in PV expected loss / 1 % increase in asset vol.	0.26	0.15	0.08

1/ Model inputs for baseline.
2/ In standard deviation of sovereign asset value.
3/ Spread in basis points.
4/ Based on a 1 percent change in sovereign asset value (e.g. from 175 to 176.75) and sovereign asset volatility (e.g. from 38 percent to 39 percent).

Appendix. Black-Scholes Option Pricing Formula in Contingent Claim Analysis

The Black-Scholes option pricing formula was originally applied to the valuation of options on traded equity and quickly spread to a variety of applications.[39] The original paper by Merton (1974) extended the option pricing formula from explicit option to implicit options in the context of corporate liabilities. He pointed out how equity can be modeled as a call option on firm assets and risky debt has an embedded put option. This work has been applied extensively (KMV 1993, Crosbie and Bohn 2001) and extended in many different directions to include multiple layers of debt and integrated with interest rate models (Cossin and Pirotte, 2001), to basket assets in multiple currencies.

A. Implied Sovereign Asset Value and Volatility

In this paper, the Black-Scholes option pricing formula is used to relate the value and volatility of domestic currency liabilities to the value and volatility of sovereign assets. The value of domestic currency liabilities as a call option on sovereign assets is,

$$V_{DCL} = V_A N(d_1) - DBe^{-r_f t} N(d_2), \qquad (A1)$$

where V_A is the value of sovereign assets, V_{DCL} is the value of domestic currency liabilities, DB is the distress barrier or value of default-free debt, r_f is the risk-free rate of interest, and t is the time to maturity on a default-free bond in years. $N(d)$ is the cumulative probability distribution function for a standard normal variable (i.e., the probability that a random draw from a standard normal distribution will be below d) where,

$$d_1 = \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r_f + \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}, \qquad (A2)$$
$$d_2 = d_1 - \sigma_A\sqrt{t},$$

and σ_A is the standard deviation of return on sovereign assets.

The Black-Scholes formula above contains two unknowns, sovereign assets and volatility of sovereign assets. The relationship between volatility of sovereign assets and volatility o domestic currency liabilities is given by,

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[39] For readers interested in a more explicit derivation of the Black-Scholes option pricing formula, see Black and Scholes (1973) and Merton (1973, 1974). While the derivations in these studies use continuous-time mathematics, Hull (1993) and Baxter and Rennie (1996) detail how binomial models can be used to develop discrete-time representations.

$$V_{DCL} = \frac{\sigma_A}{\sigma_{DCL}} \mathrm{V_A} N(d_1), \qquad \text{(A3)}$$

where σ_{DCL} is the standard deviation of domestic currency liabilities.[40] Here, $N(d_1)$ is the change in the price of domestic currency liabilities with respect to a change in sovereign assets, or $\partial V_{DCL} / \partial V_A$. This ratio is also referred to as the option *delta*. However, the main implication of the above relationship is that the standard deviation of domestic currency liabilities can be derived from historical data, including exchange rate data, and used to solve for sovereign asset volatility. Using standard iterative techniques, equations (A1) and (A3) can be solved simultaneously for the implied value of sovereign assets and sovereign asset volatility. Using this output, the precise measure of distance to distress is d_2 in equation (A2).

$$d_2 = \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r_f - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A \sqrt{t}} = \frac{\ln\left(V_A * \exp\left(\left(r_f - \frac{1}{2}\sigma_A^2\right)t\right)\right) - \ln(DB)}{\sigma_A \sqrt{t}}.$$

B. Probability of Default and Sovereign Risk Premium

The probability of default is the likelihood that future sovereign asset value will fall below the distress barrier. Therefore, computing probability of default requires calculating the cumulative normal distribution function, $N(.)$. This can be done using numerical methods or polynomial approximation. Tables that compute $N(.)$ are also found in many financial and econometric texts. Using one of these methods will yield the probability of default as,

$$\text{Risk-Neutral Probability of Default} = N(-d_2). \qquad \text{(A4)}$$

The face value of senior foreign currency debt can be derived from equation (A1) and the balance sheet relationship, $V_A = V_{DCL} + V_{FCL}$, where V_{FCL} represents the value of foreign currency liabilities.[41] Using these relationships together yields the value of foreign currency liabilities as,

$$V_{FCL} = V_A\left(1 - N(d_1)\right) + DBe^{-r_f t} N(d_2), \qquad \text{(A5)}$$

which is also equal to,

[40] See Hull (1993, p. 38).

[41] Merton (1974) derives similar measures for the pricing of corporate debt.

$$V_{FCL} = DBe^{-r_f t} - \left[DBe^{-r_f t} N(-d_1) - V_A N(-d_2)\right], \qquad (A6)$$

when modeled as the default free value minus the implicit put option (present value of expected loss). The term, DBe^{-rt}, is the distress barrier discounted to the present by the risk-free rate.

Equation (A5) can also be expressed in terms of a credit risk premium,

$$y_t - r_f = \frac{-1}{t} \ln\left\{ \frac{V_A}{DBe^{-rt}} N\left(-d_1\right) + N\left(d_2\right) \right\}, \qquad (A7)$$

where $y_t = -\left(1/t\right)\ln\left(V_{FCL}/DB\right)$. The left hand side represents the yield to maturity on risky debt less the risk-free rate of interest and is therefore equivalent to a risk premium. In addition to the risk-free rate and time, examination of equation (A7) reveals that sovereign risk premium is a function of only two variables: the volatility of sovereign assets and the ratio of the value of sovereign assets to the present value of the promised payments on foreign currency liabilities, discounted by the risk free rate. Increases in the ratio of sovereign assets to foreign currency liabilities and decreases in sovereign asset volatility both decrease the sovereign risk premium.[42]

As described in the body of the paper there is a strong relationship of the sovereign risk neutral default probabilities with the market implied default probabilities (MIDP). The risk-neutral probability of default is $N(-d_2)$. Its relationship with the estimated default probability (EDP) is,

$$N(-d_2) = N\left(N^{-1}(MIDP) + \lambda\sqrt{T}\right), \text{ where } MIDP = \frac{1-e^{-st}}{1-R} \approx EDP, \qquad (A8)$$

where λ is the market price of risk, s is the observed spread, and R is the assumed recovery rate. If we use the market implied default frequencies (MIDP) implied from observed sovereign CDS spreads as a proxy for the estimated default probability (EDP), then,

$$N^{-1}\left(N(-d_2)\right) - N^{-1}\left(\frac{1-e^{-st}}{1-R}\right) = \lambda\sqrt{T} = \frac{\mu_{Sov} - r}{\sigma_{sov}}\sqrt{T}, \qquad (A9)$$

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[42] While the BSM model assumes constant volatility, the MfRisk model is more realistic including adjustments to volatility and deviations from strictly lognormal distributions of asset value and other values such as exchange rate.

where μ_{Sov} is the return on sovereign assets, *r* is the risk-free rate, and σ_{sov} is the volatility of sovereign assets.

C. Extensions to Multiple Layer of Liabilities

Contingent claims analysis (CCA) can be extended to multiple layers of liabilities. Instead of one distress barrier there can be multiple distress barriers tied to the different layers of debt. With three layers of liabilities, the implicit options that make up the liabilities becomes, as shown in the table below: [43]

	Distress Barrier	CCA Implicit Options
Most Junior (equity-like)	Senior Plus Sub-ordinated Debt Default Barrier (DB_{Sr+Sub})	Call Option 1 (Assets, DB_{Sr+Sub}, r, t, Asset volatility)
Sub-ordinated Debt or Preferred Equity	Senior Debt Default Barrier (DB_{Sr})	Call Option 2 (Assets, DB_{Sr}, r, t, Asset volatility) minus Call Option 1 (Assets, DB_{Sr+Sub}, r, t, Asset volatility)
Senior Debt		Assets minus Call Option 2 = DB_{Sr} minus Put Option (Assets, DB_{Sr}, r, t, Asset volatility)
Total		Sum Equals Assets

The Moodys-MfRisk model uses two layers of sovereign liabilities, local currency liabilities as junior claims and foreign currency denominated debt as the senior claim. This assumption appears to be a reasonable approximation from anecdotal evidence, from the observed robustness of the model, and from the behavior of spreads during periods of stress.[44]

CCA balance sheets with three or more layers, as described above, can be constructed, calibrated and tested to refine the model further. For a consolidated CCA sovereign balance

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[43] See Cossin and Pirotte (2001) for a discussion on how the framework can handle multiple layers of liabilities or default sequences.

[44] Assuming that *all* of money and local currency debt are senior and that all of foreign currency debt is junior leads to inconsistencies. Crises resulting in depreciation of the exchange rate cause the "foreign currency junior claim" to grow large compared to domestic currency debt. This is inconsistent with the observation that CDS spreads on foreign currency debt increase with sharp depreciations. In situations of large exchange rate appreciation, usually considered beneficial from a credit risk perspective, the value of the "foreign currency debt junior claim" would be very small relative to domestic currency debt, indicating a large expected loss is associated with the domestic currency debt. Observed spreads on local and foreign currency debt during periods of stress is not consistent with assuming all local currency debt and money are senior to foreign currency debt.

sheet with three layers, the seniority structure could be: foreign currency debt as senior, part of local currency debt as subordinated and the rest of local currency liabilities as junior. Alternatively, the priority could be: part of local currency debt as senior, foreign currency debt is subordinated, followed by the remainder of local currency liabilities as the most junior claim. The three-layer model could be used to analyze the segregated *government* balance sheet. For example, the most senior claim on the government balance sheet could be foreign currency denominated debt, the subordinated claim could be local currency debt and the most junior claim could be government obligations to the monetary authorities.

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